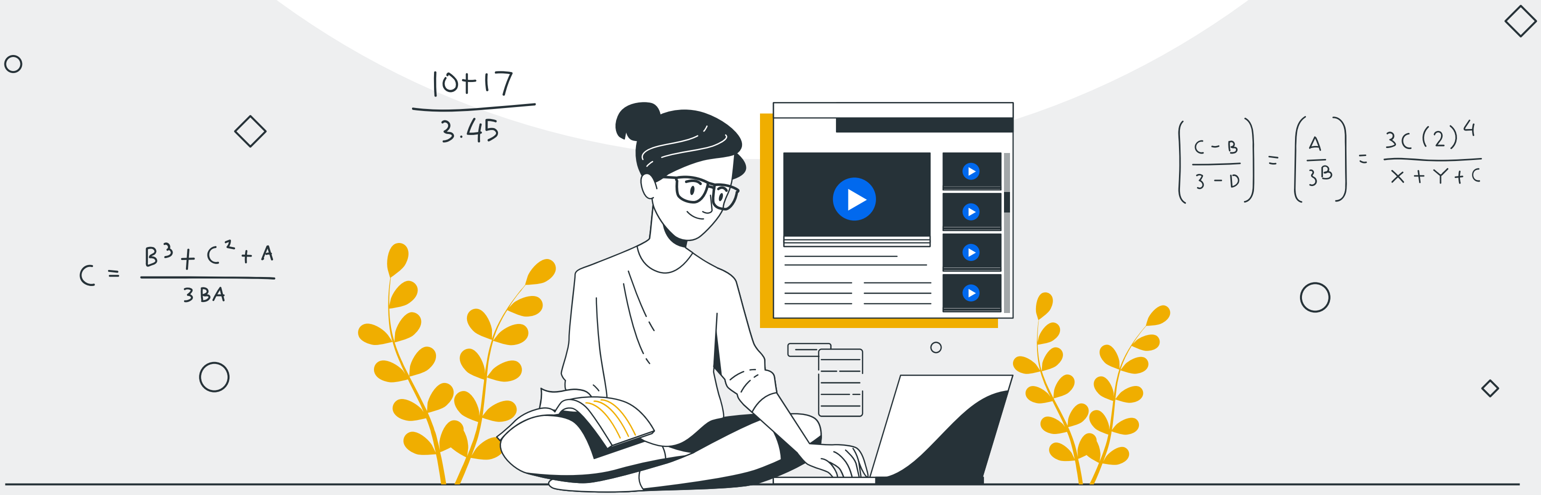
APPLICATION OF SIGNATURES FOR FORECASTING



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2021

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Review

Definition

Remark. Let us assume $X_t : [a,b] \mapsto \mathbb{R}^m$, then the signature of the path X_t is an infinite series of the iterated integrals

$$S(X)_{a,b} = (1, S(X)^1_{a,b}, S(X)^2_{a,b} \ldots, S(X)^m_{a,b}, S(X)^{11}_{a,b}, \ldots). \tag{1}$$

Examples

Review

Examples

$$S(X)^{i}_{a,t} = \int_{a<s<b} dX^{i}_{s} = X^{i}_{t} - X^{i}_{0}$$

$$S(X)^{i,j}_{a,t} = \int_{a<s<b} S(X)^{i}_{a,s}\, dX^{j}_{s} = \int_{a<r<s<t} dX^{i}_{r}\, dX^{j}_{s}$$

• Reparametrization

The signature S(X) remains invariant under time reparametrizations of X.

• Shuffle product

Theorem 2.1. We have a path $X : [a,b] \mapsto \mathbb{R}^d$, and $I = (i_1,\dots,i_k)$ and $J = (j_1,\dots,j_m)$, $(i_1,\dots,i_k, j_1,\dots,j_m \in \{1,\dots,d\})$, then

$$S^I(X)S^J(X) = \sum_{K \in I \sqcup\!\sqcup J} S^K(X)$$

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• Concatenation

Definition 2.3. Let $X : [a,b] \mapsto \mathbb{R}^d, Y : [b,c] \mapsto \mathbb{R}^d$, then the concatenation of X and Y is a path from $[a,c] \mapsto \mathbb{R}^d$:

$$(X * Y)_t = \begin{cases} X_t, & \text{if } t \in [a,b] \\ X_b + (Y_t - Y_b), & \text{if } t \in [b,c]. \end{cases}$$

Chen's identity

Theorem 2.2 (Chen's identity). As usual, let us have two paths $X : [a, b] \mapsto \mathbb{R}^d, Y : [b, c] \mapsto \mathbb{R}^d$, then

$$S(X * Y)_{a,c} = S(X)_{a,b} \otimes S(Y)_{b,c}. \tag{9}$$

Theorem

Definition 2.4. A path $X : [0,1] \mapsto \mathbb{R}^d$ is tree-like, if $\exists f : [0,1] \mapsto [0,\infty) : f(0) = f(1) = 0$ and $\forall s,t \in [0,1], s \leq t$:

$$\|X_s - X_t\| \leq f(s) + f(t) - 2 \inf_{u \in [s,t]} f(u). \tag{13}$$

Theorem 2.3. Assume $X, Y : [a,b] \mapsto \mathbb{R}^d$, then

$$\forall t \in [a,b] : X_t = Y_t \implies \forall k \in \{1,\ldots,d\} : S^k(X) = S^k(Y).$$

Theorem

Theorem 2.4 (Time reversed signature)**.** If we have a path $X : [a, b] \mapsto \mathbb{R}^d$, then the following is true:

$$S(X)_{a,b} \otimes S(\overleftarrow{X})_{a,b} = 1. \tag{15}$$

Here $\overleftarrow{X}$ is the time reversal, meaning $\overleftarrow{X}_t = X_{a+b-t}, \forall t \in [a, b]$.

Theorem

Theorem 2.3.2 (Uniqueness). *Let X be a continuous path with bounded variation. Then,*

- $S(X) = \mathbf{1}$ *if and only if X is tree-like.*
- *The signature $S(X)$ is unique up to tree-like equivalence.*

Conclusion

The path cannot be simply reconstructed from its signature in the exact speed it travels, because of the time invariance property.

However, when X does not cross itself, meaning it is a tree-like path, we can recreate the geometry of the traverse of our path.

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Application

How do we use the signature
in real life?

I would like to approximate a function, what should I do?

Taylor's theorem

But what if we do not have a differentiable function?

Approximation

$$f(X) = c_0 + c_1 S(X)^{1}_{a,b} + c_2 S(X)^{2}_{a,b} + c_{1,1} S(X)^{1,1}_{a,b} \ldots.$$

Process

How to calculate this fraction?

01 We have a data stream

02 Embed it to R^d

03 Compute the iterated integrals

04 Use the resulting set of features for analysing the data/forecasting

Example

$$\{X^1\} = \{1,2,5,6\},\ \{X^2\} = \{1,6,5,3\},$$

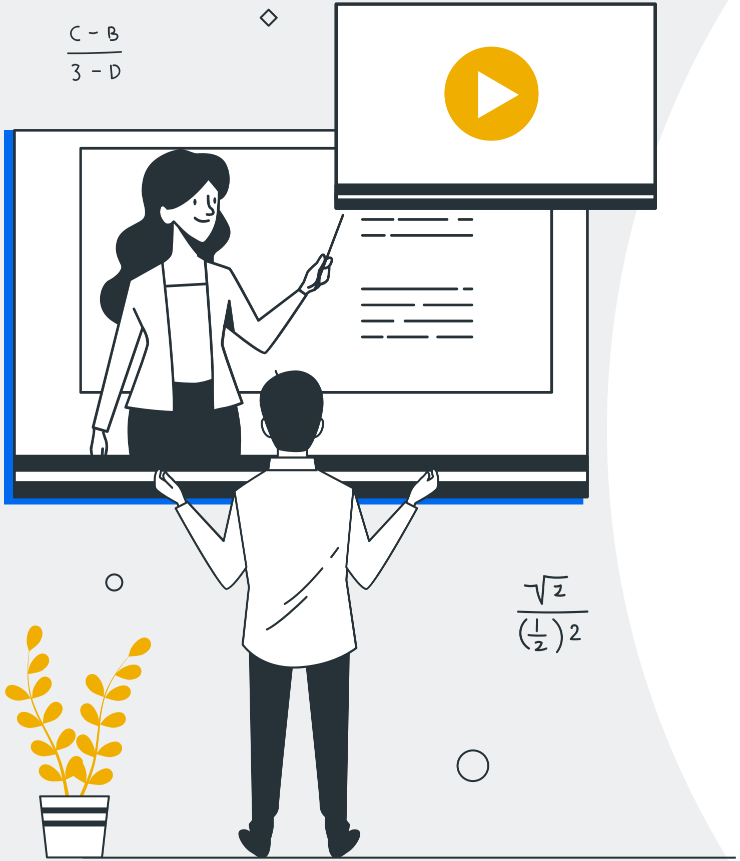
$$X^{1,lead} = \{1,2,2,5,5,6,6\}\ ,\ X^{1,lag} = \{1,1,2,2,5,5,6\}.$$

$$S(X) = \{5,2,12.5,-9,19,2\}.$$

Python

$$C = \frac{B^3 + C^2 + A}{3BA}$$

```
pip install iisignature
import iisignature as isig
import numpy as np
data= ([1,1], [2,6], [5,5], [6,3])
isig.sig(data, 2, 1)
output: (array([5., 2.]), array([12.5, -9. , 19. , 2. ]))
```



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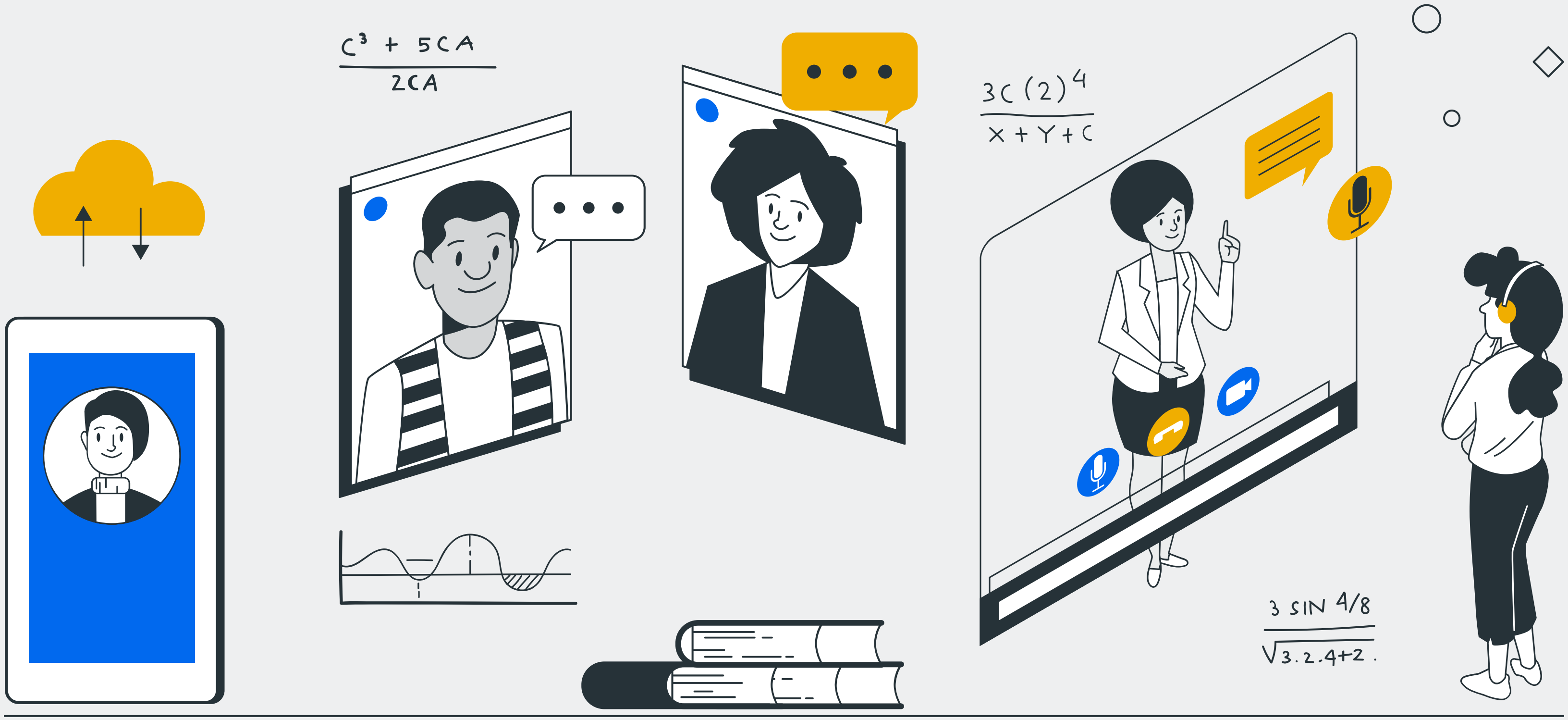

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- Maths

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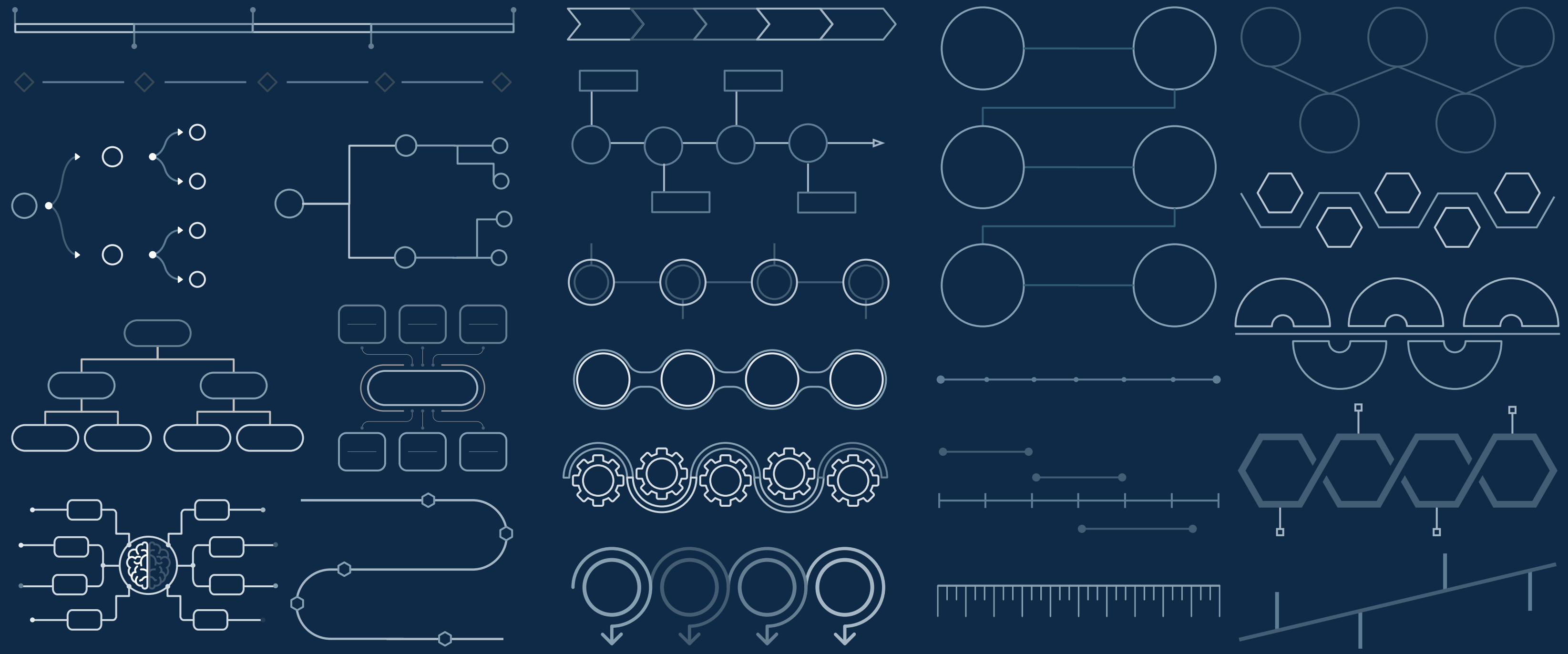

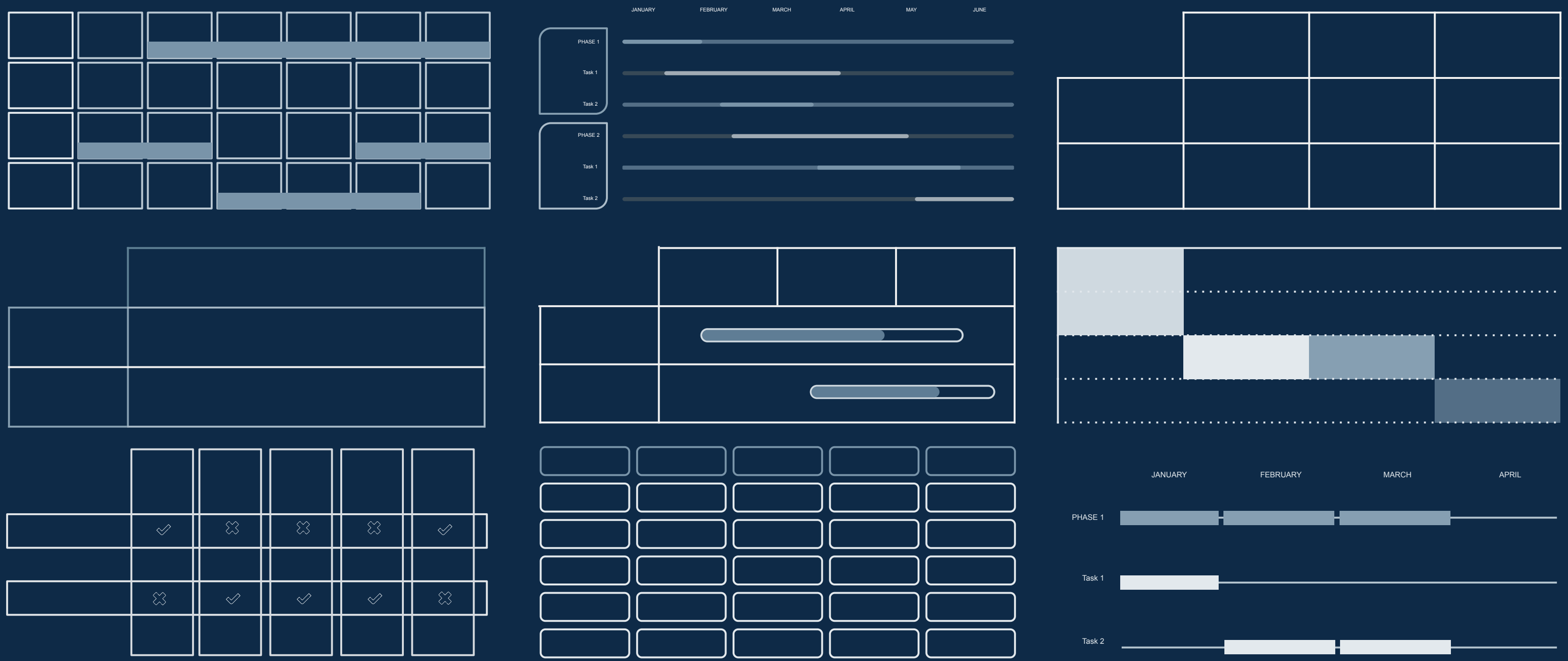
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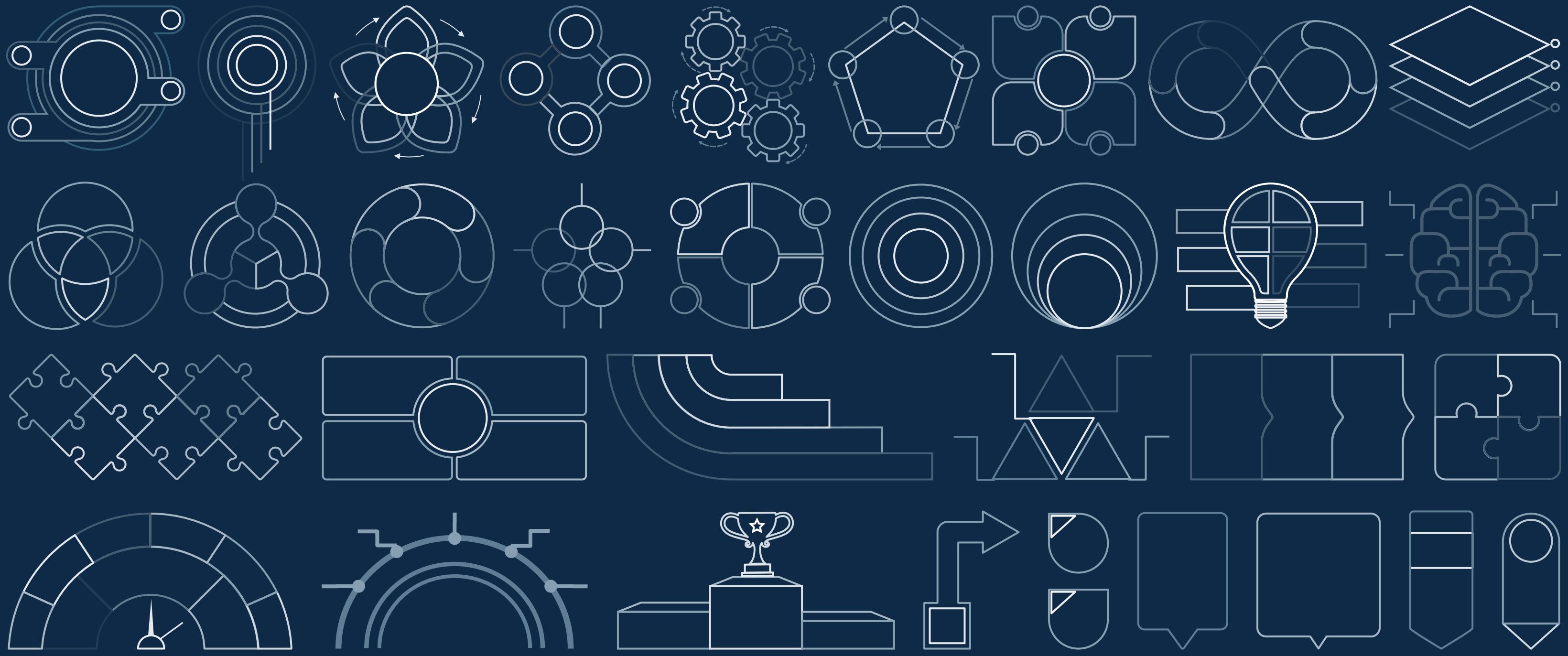

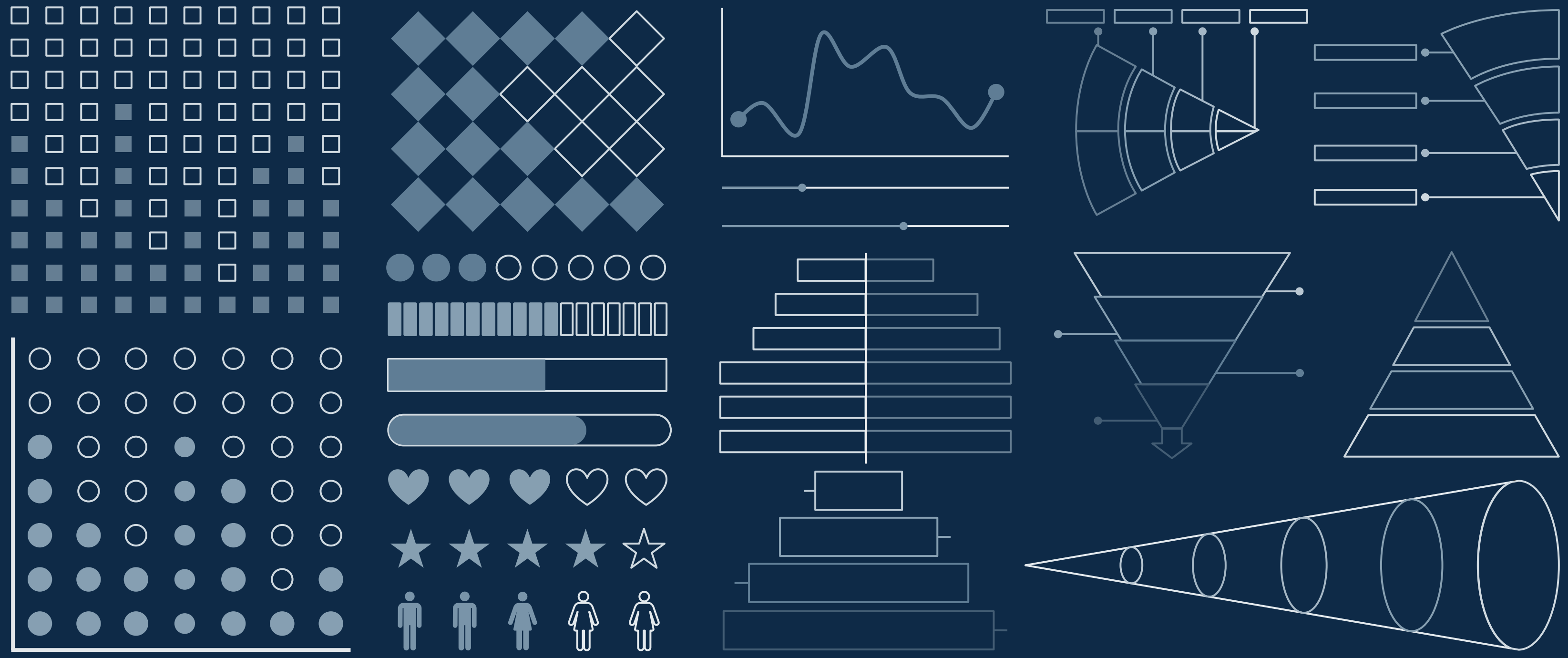

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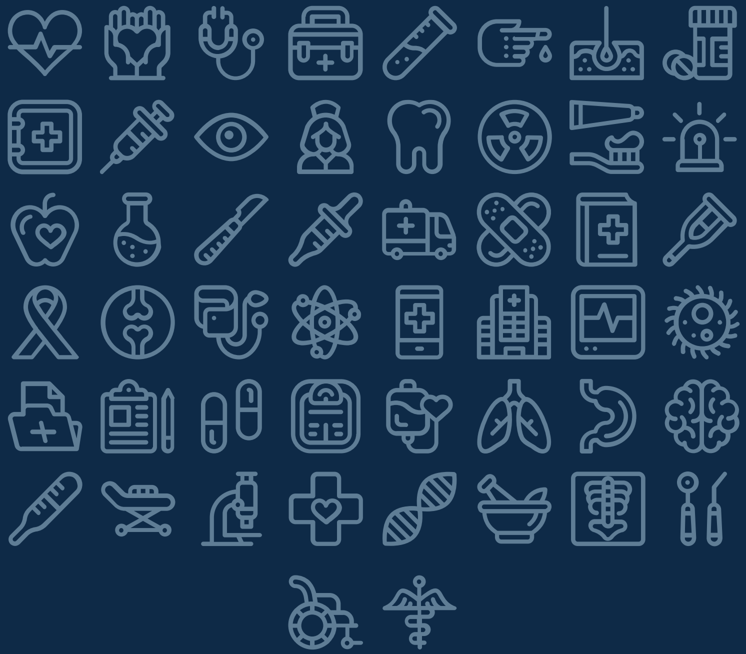

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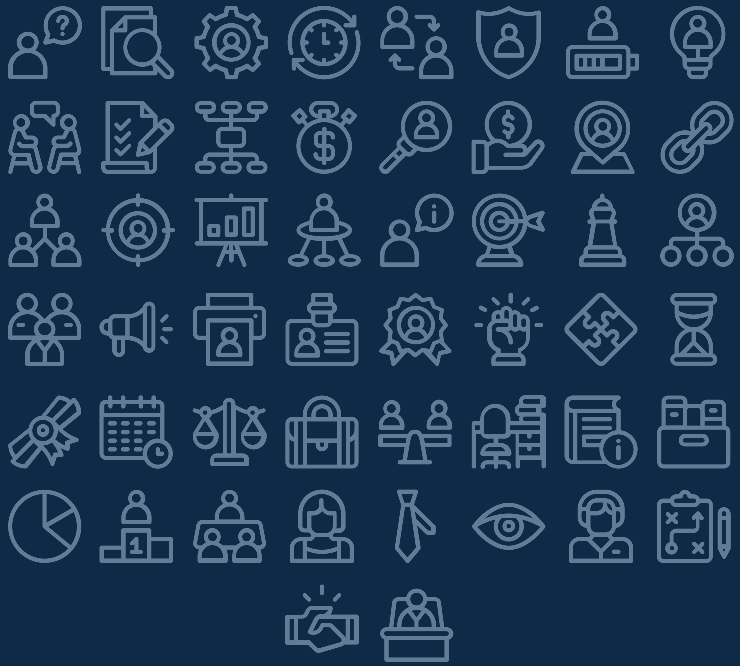

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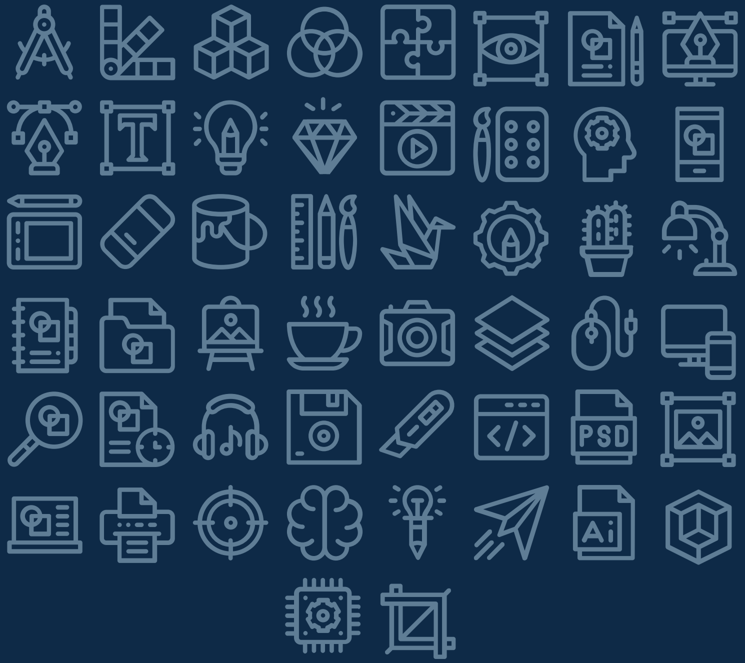

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