Graph matroid families

Szepesi Balázs
Supervisor: Imolay András

Eötvös Loránd Tudományegyetem

June 3-4, 2026

Graph Matroid Families

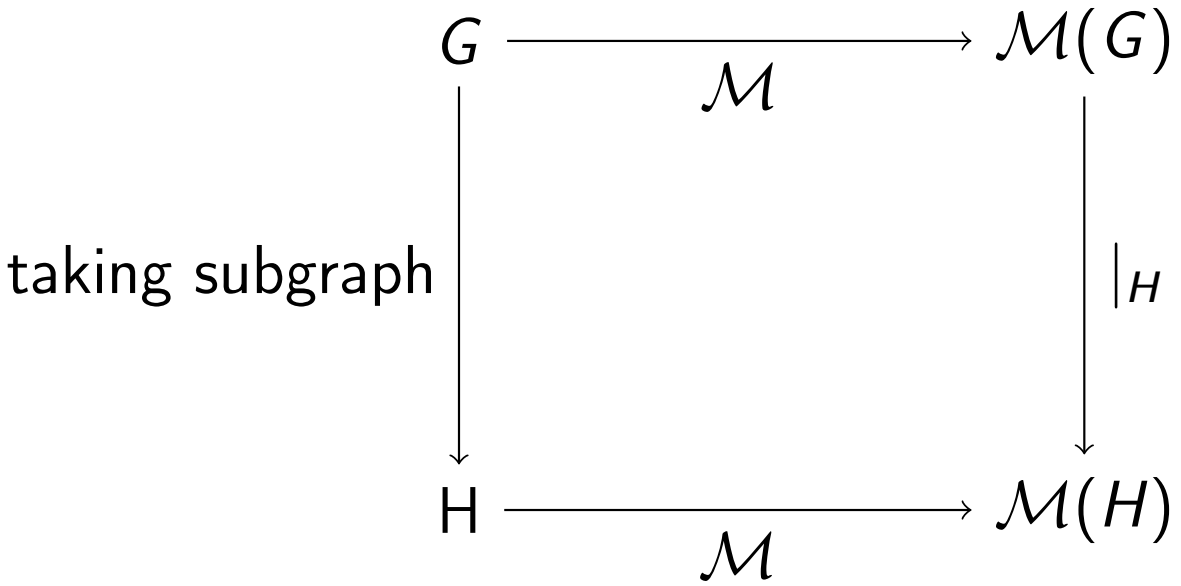
Definition

*A **graph matroid family** $\mathcal{M}$ is a family of matroids, defined on the edge set of each (finite, simple) graph in a way that is*

- *well-defined: every graph isomorphism $\varphi : V(G) \to V(H)$ induces an isomorphism between $\mathcal{M}(G)$ and $\mathcal{M}(H)$.*
- *compatible: for all subgraph H of G, $\mathcal{M}(H)$ is a restriction of $\mathcal{M}(G)$.*

Graph Matroid Families

For a given graph $G = (V, E)$ and a graph matroid family $\mathcal{M}$.



Standard Notation

- $\mathcal{M}$-independent graphs
- $\mathcal{M}$-dependent graphs
- $\mathcal{M}$-circuit graphs
- the rank function r

Dimensionality and Threshold

Dimensionality:

$$d_{\mathcal{M}} = \min\{d : \text{there exists an } \mathcal{M}\text{-circuit with minimum degree } d+1\}$$

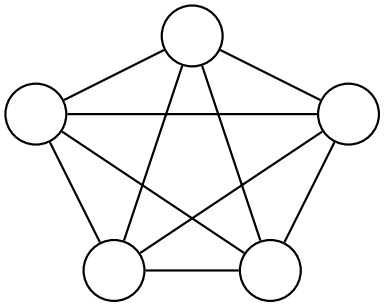
Threshold:

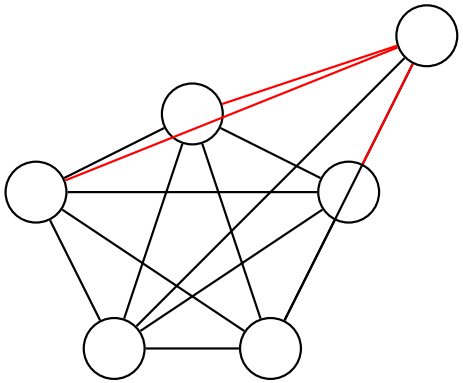
$$t_{\mathcal{M}} = \min\{|V(C)| - 1 : \ C \text{ is an } \mathcal{M}\text{-circuit with minimum degree } d_{\mathcal{M}} + 1\}$$

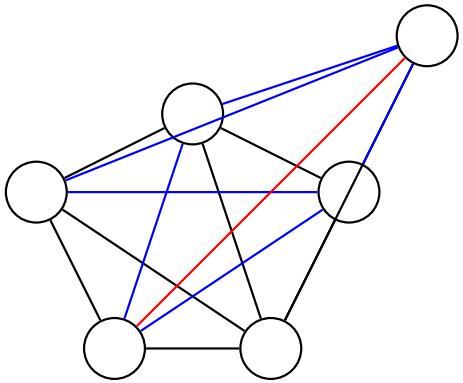
Linearity of Rank

Lemma

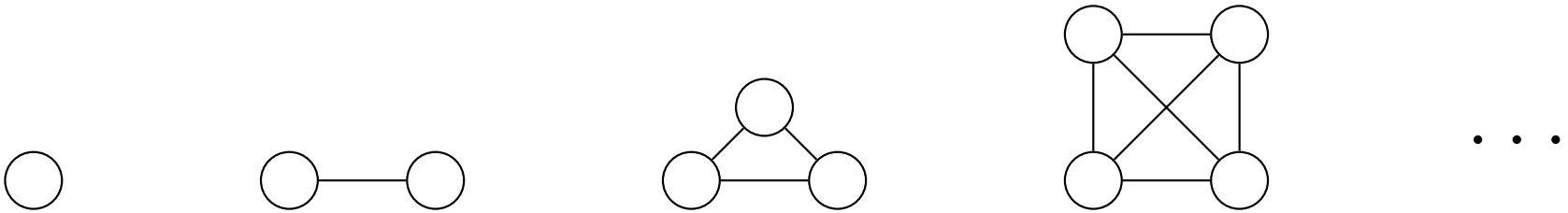
(Linearity of rank) Let $\mathcal{M}$ be a nontrivial graph matroid family with rank function r, dimensionality d and threshold t. We have $r(K_n) = d(n-t) + r(K_t)$.







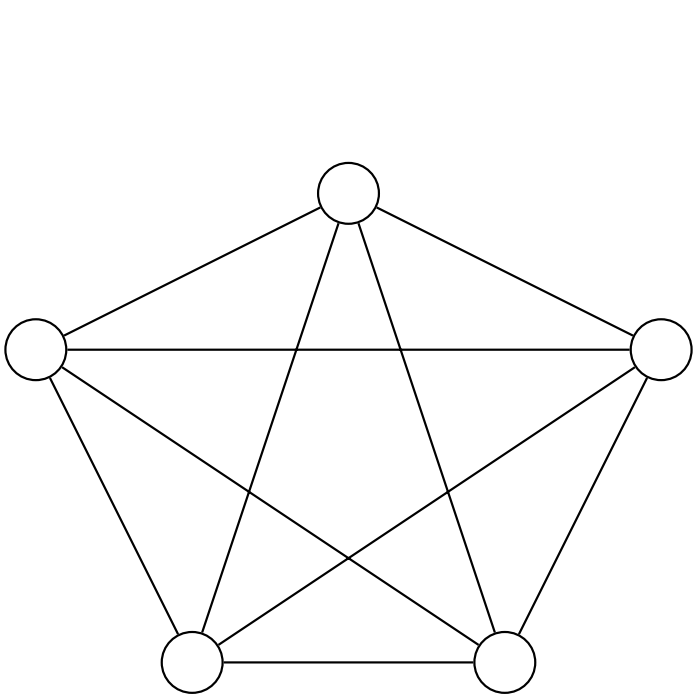
Boundedness

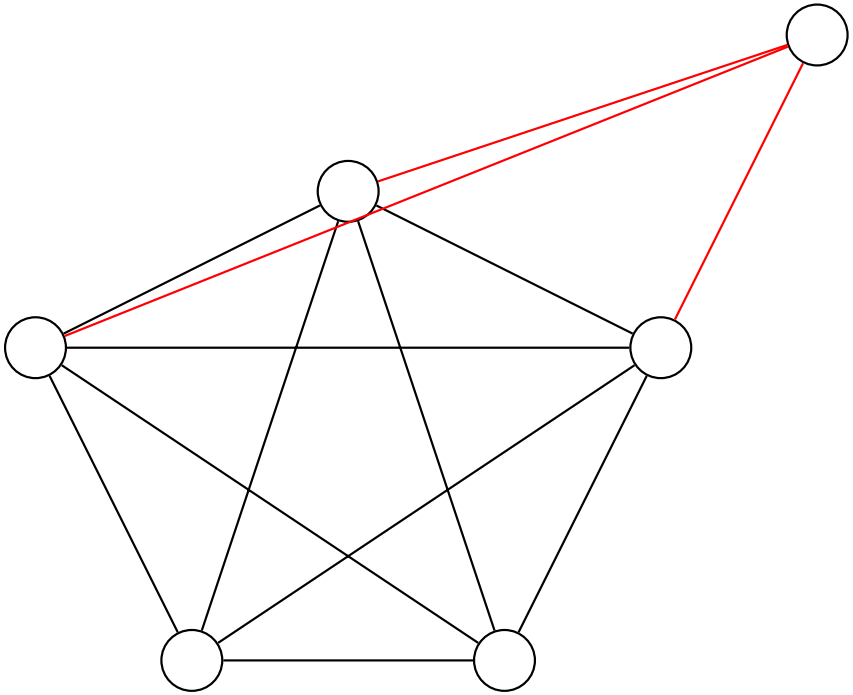


Lemma

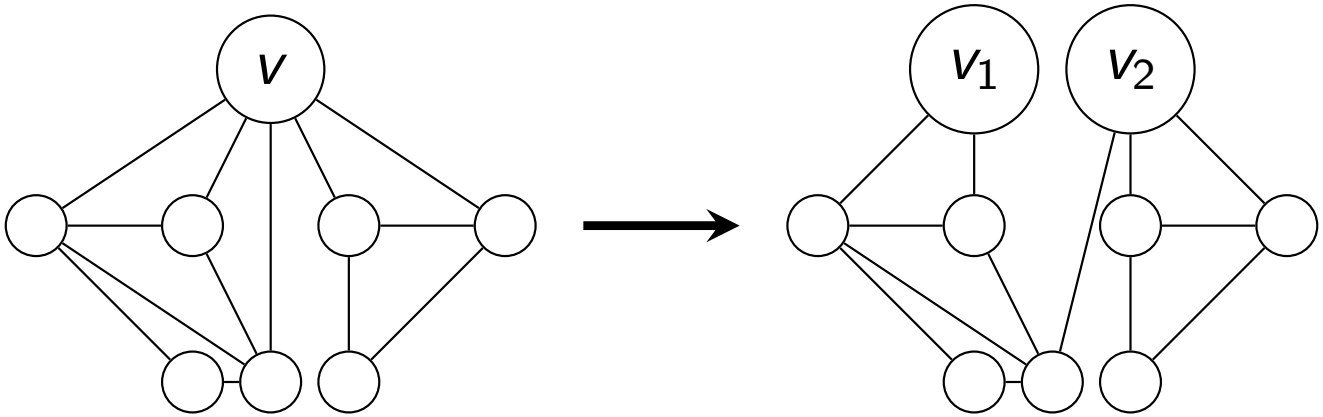
A graph matroid family $\mathcal{M}$ is unbounded if and only if every forest is $\mathcal{M}$-independent.

Vertex Addition





Splitting of a Vertex



This Semester's Question

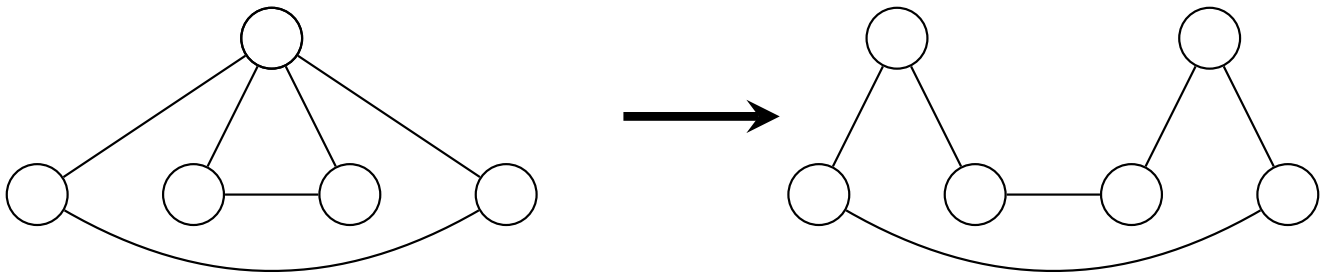
Question

Let G be an $\mathcal{M}$-independent graph, $v \in V(G)$. Can we split the vertex into two new vertices v_1 and v_2 and split its original edges among the new vertices such that the new graph is $\mathcal{M}$-dependent?

Our Example

Example

A minimal example is the graph matroid family where each graph with 6 or less edges are independent except for the C_6 (cycle of length 6) graph, and every graph on at least 7 edges is $\mathcal{M}$-dependent.



Thank you!

AI Usage

Used For	Used AI tool
Grammer correction	Writefull
Grammer correction and translation	Gemini 3.5 Flash