

# Application of arborescence packing

## Presentation 2.

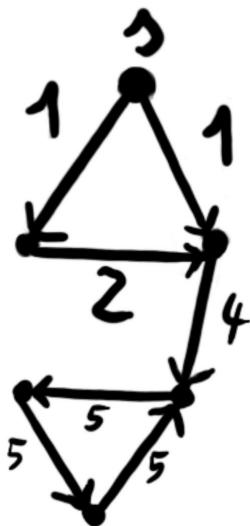
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# Introduction

Given a directed graph  $D = (V + s, A)$ .

- ▶ pre-flow:  $|\rho_D(v)| \geq |\delta_D(v)|$  for every  $v \in V$
- ▶ temporal network  $(D, \tau)$ , time label function  $\tau : A \rightarrow \mathbb{N}$
- ▶  $(D, \tau)$  is consistent: arcs of different  $\tau$ -values cannot belong to the same strongly connected component of  $D$
- ▶ time-respecting ( $\tau$ -respecting) path/arborescence:  $\tau(\vec{uv}) \leq \tau(\vec{vw})$



# Theorems

## Theorem

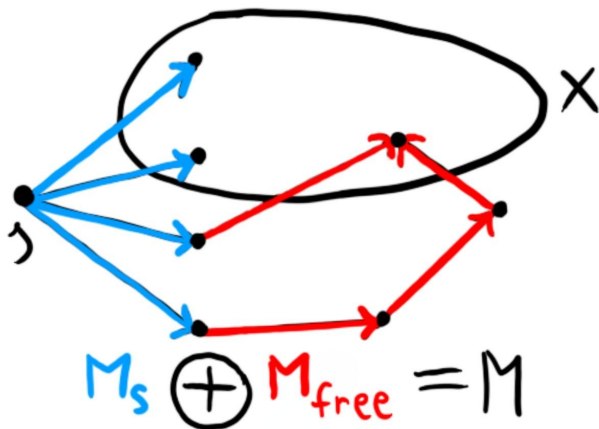
*Let  $N = ((V \cup s, A), \tau)$  be a consistent pre-flow temporal network. There exists a packing of  $|\delta_D(s)|$   $\tau$ -respecting  $s$ -arborescences s.t. each vertex  $v \in V$  belongs to  $\lambda_N(s, v)$  of them.*

## Theorem

*Let  $D = (V \cup s, A)$  be a pre-flow directed graph, and  $k := \max\{\lambda_D(s, v) : v \in V\}$ . There exists a packing of  $k$   $s$ -arborescences s.t. each vertex  $v \in V$  belongs to at least  $\lambda_D(s, v)$  of them.*

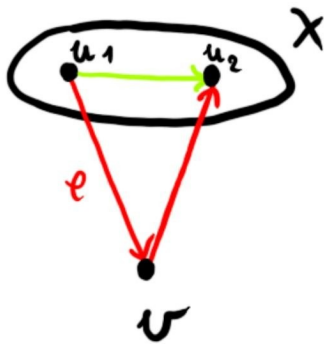
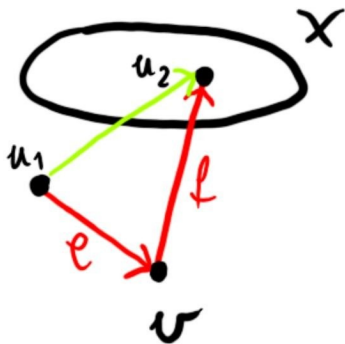
## The matroid

- ▶  $\lambda_{\mathcal{M}}(s, v)$ : max. rank of the root-edges of the arc-disjoint  $sv$  paths
- ▶  $\varrho^r(X)$



# Split off

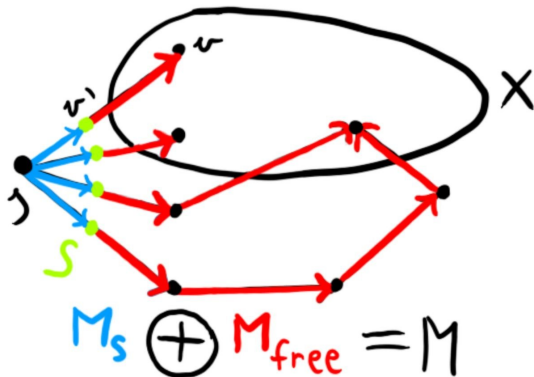
- ▶ split off edges  $e$  and  $f$



## New matroid

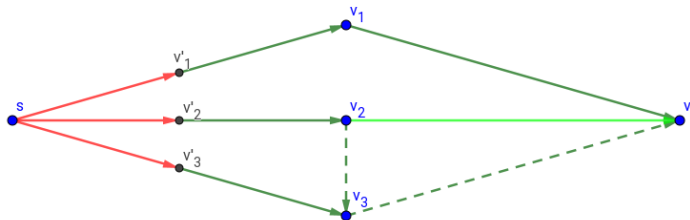
*Extension:* In this matroid  $\mathcal{M}$  we can pack arc-disjoint  $s$ -arborescences where every vertex will be covered by at least  $\lambda_{\mathcal{M}}(s, v)$  arborescences.

Suppose:  $e, f$  edges of  $v^*$  ( $\lambda_{\mathcal{M}}(s, v^*)$  is min.) can be split off, s.t.  
 $\lambda_{\mathcal{M}'}(s, v) = \lambda_{\mathcal{M}}(s, v) \forall v \in V - v^*, \lambda_{\mathcal{M}'}(s, v^*) \geq \lambda_{\mathcal{M}}(s, v^*) - 1$



## Future plans

- ▶  $\lambda_{\mathcal{M}}(s, v) = \min\{\varrho^r(X) : X \subseteq V, v \in X\}$  ( $\approx$  Menger's theorem)
- ▶  $\varrho^r(X) \geq \lambda_{\mathcal{M}}(s, v)$
- ▶  $\lambda_{\mathcal{M}}(s, v) := \min\{\varrho^r(X) : X \subseteq V \cup S, v \in X\}$
- ▶  $\lambda_{\mathcal{M}}(X) := \max\{\lambda_{\mathcal{M}}(s, v) : v \in X\}$
- ▶  $X \subseteq V \cup S$  tight:  $\lambda_{\mathcal{M}}(X) = \varrho^r(X)$



# Using of AI

During my work I only used AI (ChatGPT) as an English dictionary and I asked some Latex codes too.

**Thank You for Your attention!**