Power Spectral Analysis of Seiches in Lake Fertő

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May 17, 2026

1 Introduction

Basin-scale oscillations, also known as seiches, are standing waves in enclosed or partially enclosed bodies of water. In lakes, they are formed when the effect of external forces (most often wind and changes in atmospheric pressure) cause the water in the basin to oscillate for longer periods of time. The ultimate goal of this project is to identify and characterize seiches in Lake Fertő using time series data from multiple shoreline gauge stations. By applying a combination of spectral, cross-spectral, and statistical methods, the project seeks to determine which oscillatory modes are detectable in observations, how their properties vary spatially along the shoreline, and how they relate to the physical characteristics of the lake. A particular challenge is that Lake Fertő is considerably smaller and much shallower than the large lakes typically analysed in the seiche literature, with considerably dense vegetation, implying potentially stronger damping and less persistent oscillations.

The main objective for the second part of this project work was to implement additional methods of signal processing which, together with the results of the first semester, give a more cohesive picture of the seiches of lake Fertő. In particular, the Power Spactral- and EOF analyses were supplemented with cross-spectral coherence analysis between the stations and wavelet analysis, based on the reference study ([Man20]). Wavelet transforms, together with the results of earlier methods, were investigated more closely around known storm- or otherwise unusual weather conditions.

2 Theoretical Background

As per the referenced study [Man20], at basin scale, lake seiches can be approximated as long, shallow-water waves. In their simplest form, the dynamics of a single basin mode may be reduced to that of a forced, linearly damped harmonic oscillator. The governing equation of this "first basin mode" can be written as

$$\frac{d^2A}{dt^2} + 2r\frac{dA}{dt} + \omega_n^2 A = f(t), \tag{1}$$

where A is the modal amplitude, r is the damping coefficient, ω_n is the natural seiche frequency, and $f(t)$ represents the external forces. Damping r is to be calculated via the quality factor Q, using the relation $r = \frac{\omega_{peak}}{2Q}$. Q is the factor that measures the ratio of stored energy to energy lost per oscillation cycle, and is often calculated as the ratio of the resonance frequency divided by the width of the frequency region falling within 3db of the response at resonance frequency. A relatively high Q indicates that the oscillations persist for longer periods of time, while a low Q is the sign of a more damped system where the oscillations decay quickly. In the case of Lake Fertő, the initial expectation is a low Q, as the average water level is shallow and vegetation is high.

2.1 Power Spectral Analysis

Power spectral density (PSD) analysis quantifies the distribution of variance in a time series as a function of frequency. As per [Sem11], the PSD may be estimated from the squared magnitude of its Fourier transform:

$$PS(\omega) = |Y(\omega)|^2 \tag{2}$$

where

$$Y(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{3}$$

if f(t) denotes the sampled data. Since water level measurements are on a discrete time scale (15 minutes for the data used in this project), in practice an efficient implementation of the discrete Fourier transform, the fast Fourier transform can be used.

Deriving the power spectral density of the sampled data is a powerful tool here as in an ideal case the frequency of the first basin mode can be easily read from the resulting periodograms, as it will form a peak in resonance at all the measuring points. As multiple basin modes exist, the resonance peak at the lowest frequency should be identified as the first basin mode in the relevant frequency range. A decent estimate fot the longest natural period T that can aid the identification process can be computed using Merian's equation:

$$T = \frac{2L}{\sqrt{gh}} \tag{4}$$

where L and h are the length and average depth of the body of water, and g is the acceleration of gravity.

2.2 Cross-spectra and Coherence

As basin-level oscillations are inherently spatially coherent, periodograms from individual measuring stations are unable to fully describe them. Cross-spectral analysis can provide a measure of coherence between pairs of measuring stations on the frequency domain. Taking two time series $y_1(t)$ and $y_2(t)$, their cross-spectra can be calculated as

$$\gamma_{12}^2(\omega) = \frac{|S_{12}(\omega)|^2}{S_{11}(\omega)S_{22}(\omega)} \tag{5}$$

where S_{11} is the one-sided spectrum, S_{12} is the one-sided cross-spectrum, and is calculated via the Fourier transform $Y(\omega)$ of time series $y(t)$ as

$$S_{12}(\omega) = \frac{2}{N\Delta t}[Y_1^*(\omega)Y_2(\omega)], \tag{6}$$

N denoting the number of data points. Coherence can be used to verify the existance of a basin-wide mode. In the referenced study [Man20] it is calculated for all stations against a fixed reference station, and all coherence figures show a peak at the theoreticised first basin mode frequency.

2.3 EOF

Empirical Orthogonal Functions (also known as PCA or SVD) can be used to decompose the variance of time series data into modes with relative contributions from each station. When ordered by the measure of explained variance, the coefficients of the first EOF are often similar at all stations (representing seasonal or longer term variability of the entire lake level), while the coefficients of the second EOF are often ordered as the stations themselves are ordered along the dominant axis of the lake, with negative coefficients on one side, and positive coefficients on the other. This behavior suggests that the second EOF represents the basin-wide seiche mode.

2.4 Wavelet Analysis

The wavelet transform of a time series produces the amplitude and phase of the seiche for every station at every point in time. The role of this analysis is to localise wave packets in the signal both in time and frequency. Afterwards, the transform can be investigated near known disturbances, and the amplitudal reaction at the seiche frequency can confirm its existance. A limitation of this methodology that must be mentioned is the Heisenberg uncertainty principle: The temporal and frequency resolution are inversely proportional, meaning that their proper ration must be selected carefully to achieve desired accuracy in both domains.

Closely following the methodology of the reference study, the 'cwtft' Matlab function was used for the computation of the transform, which is a fast Fourier transform based method, together with Morlet wavelets.

3 Initial Results

3.1 Data

This project work uses measurements from 8 measuring stations along the shoreline of lake Fertő, with data points from between 2009 and 2015, sampled at 15 minute intervals. The analysis was carried out in MATLAB. The measurements of station Fertőrákos were omitted, as they contained high levels of missing values. The data used for the analyses was first detrended. De-tiding, as in the reference study, proved unnecessary, as the effect of a tide was not detectable on the periodograms, due to the small size of the lake. Some figures of the analysis are presented below.

3.2 Power Spectral and Cross-Spectral Analysis

Power spectra were computed using the direct periodogram approach and expressed in cycles per day. Spectral estimates were stabilized through frequency-domain band-averaging and examined for the full record, as well as on a year-by-year and seasonal basis. These analyses were carried out for all measuring stations(1). Unlike the referenced study, the power spectra did not show clear resonance peaks and remained quite noisy even after high levels of band-averaging. The only peak identifiable at all stations was around the 24 cpd frequency, suggesting hourly oscillations, but still was not clear enough to be conclusive.

The magnitude squared coherence of cross-spectra was also computed between a chosen reference station and all other stations(2). These figures were computed based on the entire time interval, as the data contained no missing values in the investigated time frame. This analysis helped in the understanding of the phase relation between these stations. If the seiche frequency could have been pin-pointed by the PSD analysis, the phase relation at the frequency could have revealed the spatial structure of the seiche. Since the seiche frequency is not clearly pin-pointable, the coherence figures are instead used to identify further possible frequencies. Those frequencies, where the coherence between all station pairs is high, and the phase relation is roughly aligned with the expectation based on the lake shape (where stations on the same longitudinal side of the lake being in similar phase, while stations opposite to each other being in opposite phase) are good candidates to explore in further analyses. Based on this, however, clear seiche frequencies still cannot be pointed out; there is no undeniably clear maximum of coherence values (as in the reference study) that apply for all station pairs.

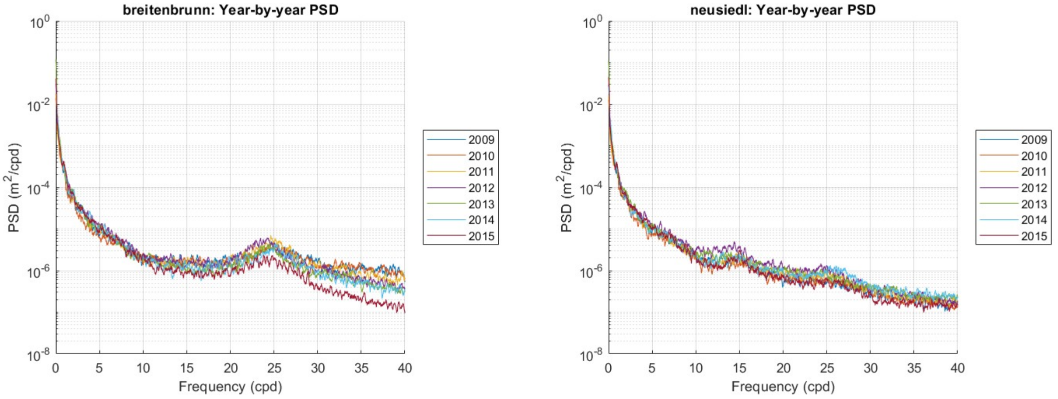

Figure 1: PSD of two stations, separating yearly data

3.3 EOF

EOF analysis revealed a dominant first mode explaining the majority of variance, with nearly uniform loadings across all stations(3). This mode is interpreted as basin-wide, low-frequency variability associated with seasonal and/or longer-term water-level changes. The second EOF exhibited a systematic sign change between the northern and southern shores, with intermediate stations ordered approximately monotonically in between, consistent with the spatial structure expected for a longitudinal basin-scale oscillation (and the results of the referenced research). A significant difference, however, is that the first EOF dominates the variance of data: it already explains around 90% of lake level variability. This aligns well with the setting: Lake Fertő is very

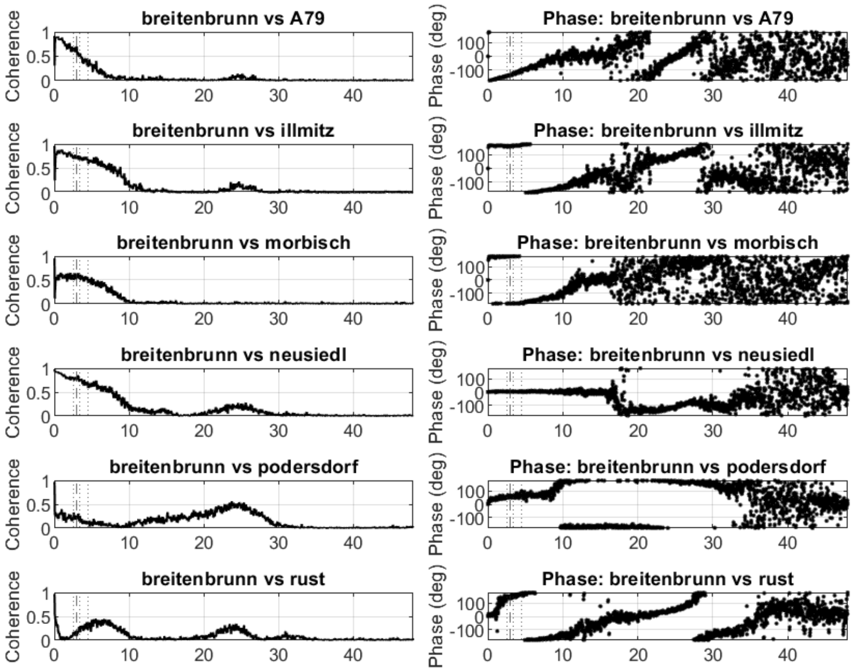

Figure 2: Coherence of cross-spectra for all stations paired with reference station

shallow, and its water level movements are generally dominated by overall lake level fluctuations, while the effect of seiches should account for significantly less variability.

Despite this physically plausible spatial pattern, the associated temporal coefficients and station-level PSDs did not exhibit a sharp, persistent spectral peak. This suggests strong damping and intermittent excitation of the oscillation; again, in agreement with expectations for a shallow, vegetation-rich lake such as Fertő.

Power spectra of individual EOF modes were investigated as well, in the hope of identifying a more clear oscillatory peak when only plotting the second EOF mode; this however did not prove more insightful either, most likely due to the strong damping in the system.

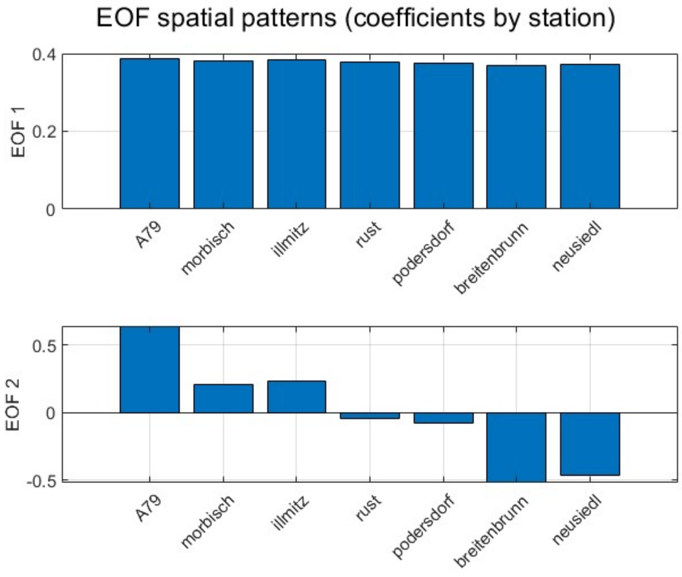

Figure 3: Coefficients of the first two EOF by station.

3.4 Analysis of especially affected periods

As seiches are naturally formed by weather effects, their identification should be made easier by investigating time periods, where unusual weather effects forced the lake, and therefore put the seiche in motion. In this spirit, several heavy storm- or wind events were selected inside the studied time frame, and the above analyses were computed in each case for a time window of 20 days around the weather event.

While the PSD periodograms were noisy and unclear in most of the cases, the coherence tables for these smaller time intervals showed much higher coherence values and clearer common peaks. An example of inter-station coherence during an unusual weather event around 2013 December 24. is shown on figure 4. The spectral peaks present on all these event-based coherence figures are around the 5-6 CPD, 12-13 CPD, and 31-32 CPD marks, which, solely based on these, might correspond to seiche modes. Their absence in previous investigations could be explained by the characteristics of the lake: its reaction to weather events might be insignificant when averaged through longer periods of time, as the following oscillations are strongly damped and do not persist for elongated time intervals. Only when the effect of these events is closely investigated is when these frequencies can be pin-pointed.

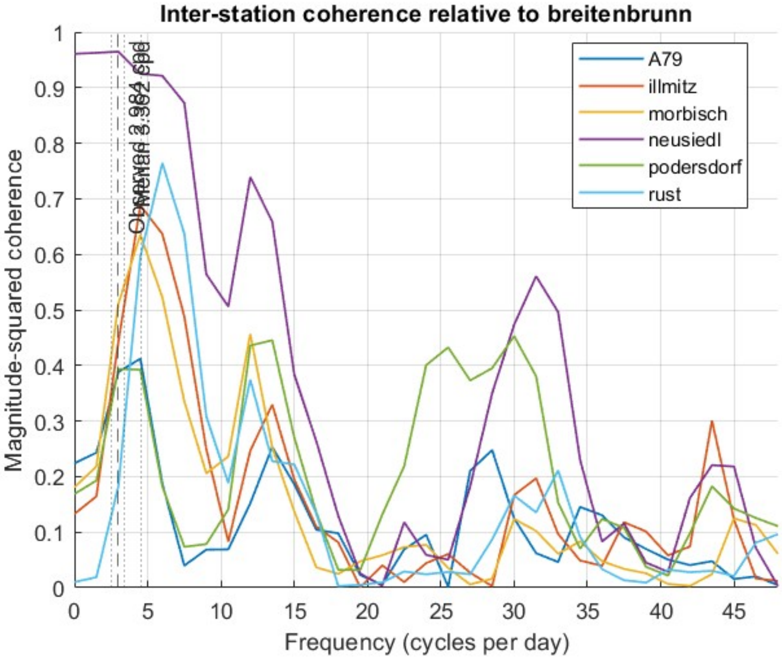

Figure 4: Coherence of cross-spectra for all stations paired with reference station

3.5 Wavelet Analysis

The wavelet transform of the entire time series was also investigated based on known weather events. For time windows around each of these events, different frequency ranges were investigated to identify frequencies where a clear response to sudden weather forcing could be detected. These frequencies coincided for three stations: the "A79", "breitenbrunn" and "neusiedl" gauges showed response to these events on similar frequencies, while the other four stations showed little to no response in these frequencies. This can be explained by the spatial location of the gauges: the other four stations are located close to the longitudinal center of the lake, and are therefore close to the theoretical nodal line of the first seiche mode.

The wavelet transform of the "breitenbrunn" lake level data in two differetn stormy periods can be found on figure 5. Due to a not yet identified (but obviously present) error in the wavelet transform implementation the scale of these figures is currently incorrect, and therefore non-real frequencies are displayed. This error will be fixed as shortly as possible, but the reaction of these frequencies to the weather event is still nicely visible.

4 Future Work

In the following semester, the wavelet analysis calculations will be re-checked and fixed, while additional data processing techniques will be explored based on the reference study. These include the modal reconstruction accross the lake basin and the development of a linear response model to outside forcing (Green function). Additionally, previous work will be meticulously checked against other studies on lakes with comparable dimensionality, from where other methodologies might be implemented as well.

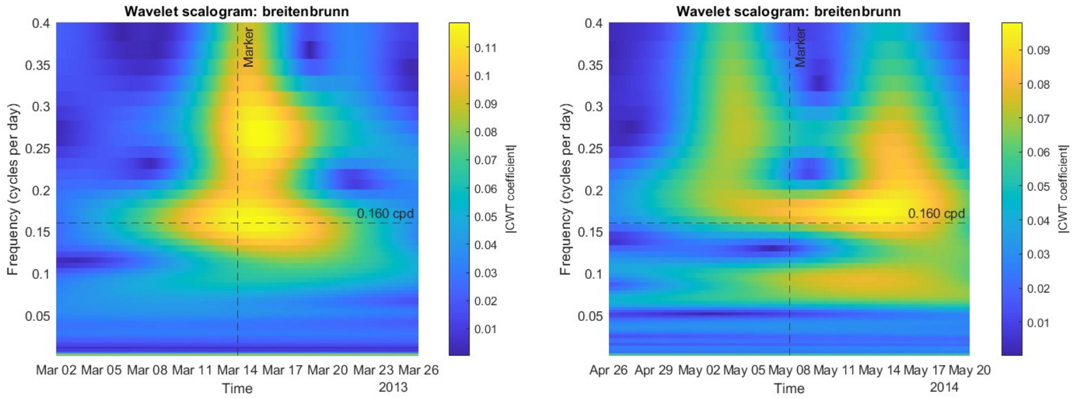

Figure 5: PSD of two stations, separating yearly data

References

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