# Power spectral analysis of seiches in Lake Fertő

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04-06-2026

Project Work II.
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# Agenda

# Motivation

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**Seiches**

- Standing waves formed by external forces (wind, atmospheric pressure)
- The oscillation of water persists for longer periods of time
- Seiche periods can only be roughly estimated from theory alone

**Relevance**

- Flushing of vegetation around the lake
- Damage to shoreline stuctures

**Lake Fertő**

- Small basin, shallow water, high level of vegetation
- Strong damping, less persistent oscillations expected

# Goal of the project

- Understand mathematical tools and frameworks of the reference study
- Implement analyses, adjust methodology as needed
- Interpret the results, develop an understanding of seiche modes

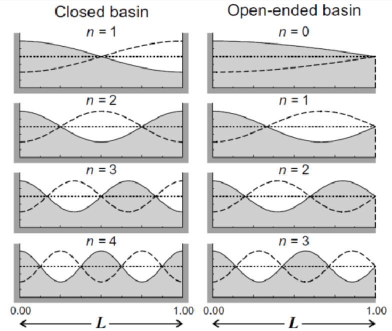



**Figure 1:** Basin modes in closed and open-ended basins (source: Rabinovich, 2009)

# Theoretical background

# Forced Damped Harmonic Oscillator

Overarching equation: **Forced, damped harmonic oscillator**

The evolution equation of amplitude $A$ can be given by

$$\frac{d^2A}{dt^2} + 2r\frac{dA}{dt} + \omega_n^2 A = F \tag{1}$$

- $r$ is the damping coefficient
- $\omega_n$ is the natural seiche frequency
- **F** is the external forcing

$r$ and $\omega_n$ are to be estimated using power spectral analysis

$r$ can be estimated through its relation to the Q factor:

$$r = \frac{\omega_{peak}}{2Q} \tag{2}$$

$$Q = \frac{\omega_{peak}}{\Delta\omega_{-3dB}} \tag{3}$$

# Power Spectral Analysis

To determine $\omega_{peak}$, we use power spectral analysis.

We convert the time-dependent measurements to the frequency domain

$$PS(\omega) = |Y(\omega)|^2 \tag{4}$$

here, $Y(\omega)$ denotes the Fourier transform:

$$Y(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{5}$$

while $f(t)$ is the lake level data from measurements.

Practice:

- Data is on a discrete scale
- Fast Fourier Transform is used
- Hope: an easily detectable peak at the frequency where energy is the strongest

# Empirical Orthogonal Functions

- More generally called Principle Component Analysis
- Used to capture the dimensions in the data that make up most of the variance
- Decomposition of data into EOF coefficients and Principle Component time series
- Here: used to deconstruct lake level variability into modes that explain a large part of this variability
- Primary goal: confirm the existence of a basin wide seiche

# Cross-Spectral Analysis, Coherence, Phase

The seiche is inherently spatially coherent

Magnitude squared coherence shows the relation of two power spectra at each frequency

$$\gamma_{12}^2(\omega) = \frac{|S_{12}(\omega)|^2}{S_{11}(\omega)S_{22}(\omega)} \quad (6)$$

$$S_{12}(\omega) = \frac{2}{N\Delta t}[Y_1^*(\omega)Y_2(\omega)] \quad (7)$$

The phase relation between the two time series shows the phase lag at a given frequency:

$$\phi_{12}(\omega) = arg(S_{12}(\omega)) \quad (8)$$

Hope: high inter-station coherence at seiche frequency, phase values confirming the theoretical mode shape

# Wavelet Analysis

- Used to produce complex seiche amplitude time series
- Localizes wave packets in time and frequency
- Hope: visible increase in amplitude at seiche frequency, around or after external forcing events

# Merian's equation

The theoretical period of the first basin mode can be estimated

$$T = \frac{2L}{\sqrt{gh}} \tag{9}$$

Where

- $T$ is the longest natural period
- $L$ is the length, $h$ is the depth of the body of water
- $g$ is the acceleration of gravity

In the case of Lake Fertő, the theoretical longitudinal seiche is around 5-7 hours (∼4 cpd)

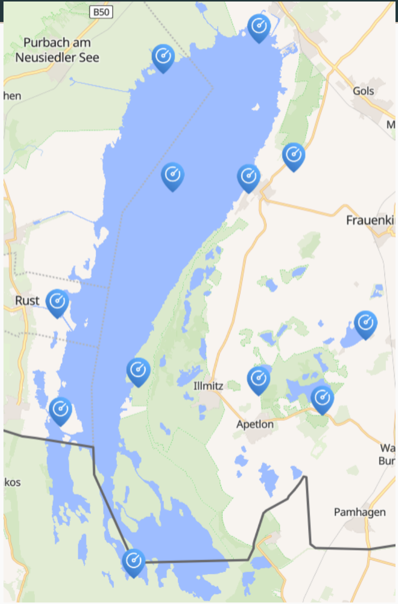
Highly influenced by basin shape, change in depth of water (the equation holds for rectangular basins)

Can be used as a reference point

# Results

# Data Preparation

- Data used from 7 (+1) stations around Lake Fertő
- Data from the Fertőrákos gauge initially omitted
- Water level data taken in 15 minute intervals between 2009 and 2015
- Before analysis the data was detrended, tide removal is deemed unnecessary
- Prior to EOF analysis, the mean water level at each station was substracted
- Different levels of band-averaging used to smooth out data



# Results: EOF

Expectation:

- First principal component determining lake-wide water level variations
- Second PC characterising the primary seiche (first basin mode)
- The first two PC-s explain most of the variance

# Results: EOF

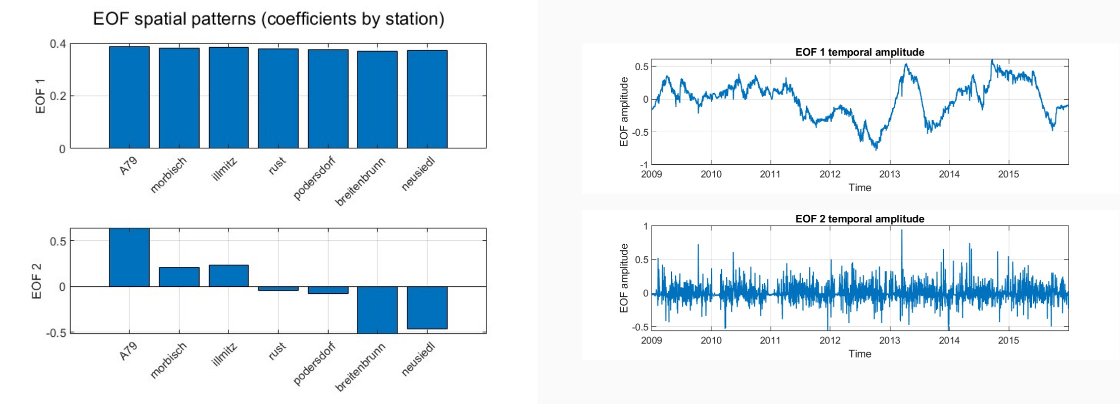
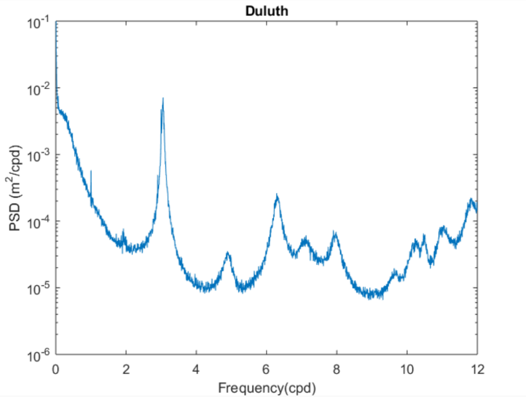



**Figure 2:** Coefficients of the first two EOFs by station, along with the time series of the first two principal components. These together explain over 95% of lake level variance

# Results: PSD

- PSD calculated for all stations across the entire time period, yearly, and seasonally as well
- Band-averaging to negate noise
- Expectation: Clear peak at resonance frequency



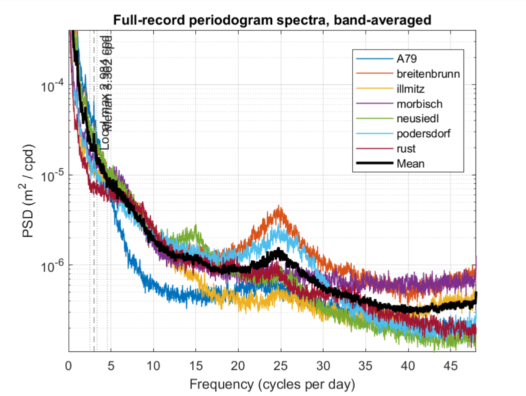




**Figure 3:** PSD of the reference study for Lake Superior vs. PSD for Lake Fertő at all stations

# Results: Coherence

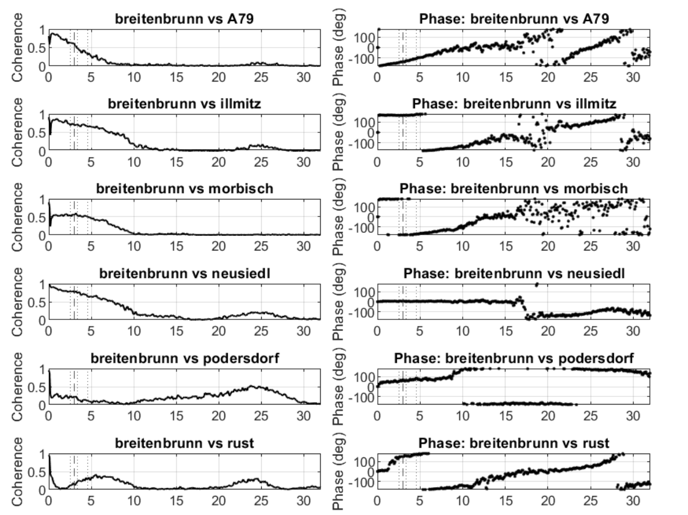



**Figure 4:** Magnitude squared coherence between all stations and reference station Breitenbrunn

# PSD and Coherence around storm events

- Highly damped system expected; the second EOF explains low variance
- PSD and Coherence averages out the energy carried by a frequency

$\Longrightarrow$ Closer analysis around stormy periods might yield clearer results

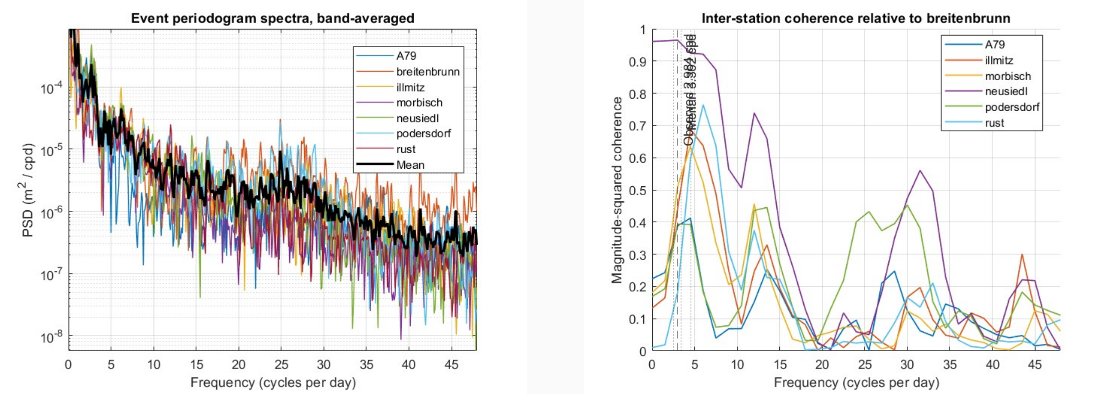



**Figure 5:** Power spectra of all stations during the storm period. Coherence between reference (Breitenbrunn) and all other stations

# Wavelet analysis

Wavelet transforms around storm events were also analyzed at candidate frequencies

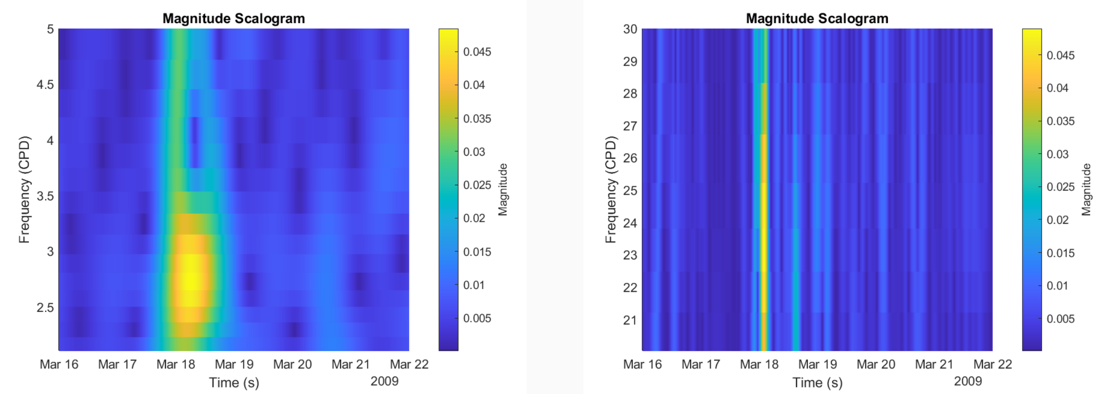



**Figure 6:** Wavelet transform of the time series around a windy storm (March 18), around 3 and 24 cpd. These may represent different seiche modes

Oscillations around 3 cpd appear strongest for the stations at the longitudinal ends of the lake, while oscillations around 24 cpd are present at all stations.

# Plans for the following work

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- Acquire and analyse wind data from the investigated time period
- Analyze storm events of similar wind direction
- Generate spectrum of wind levels, coherence with lake level spectrum

# Thank you for your attention!

# Usage of Artificial Intelligence

During my project work, I used ChatGPT to help with some unclear parts of code implementation.

# Sources


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