

# Spectral Analysis of Seiche Phenomena in Lake Balaton II.

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## 1 Introduction and Theoretical Background

### 1.1 The Seiche Phenomenon in Lake Balaton

Hydrodynamic macro-oscillations, commonly known as seiches, frequently occur in shallow, elongated lakes like Lake Balaton due to meteorological forcing. The theoretical free oscillation period ( $T_m$ ) for a rectangular basin of uniform depth can be calculated using the Merian formula:

$$T_m = \frac{2 \cdot L}{m \cdot \sqrt{g \cdot h}} \quad (1)$$

Where  $L$  is the basin length,  $h$  is the water depth,  $g$  is the acceleration due to gravity, and  $m$  represents the mode index (number of nodal lines). For Lake Balaton's actual geometry, the fundamental longitudinal mode ( $m = 1$ ) is documented to range between 11 and 12.5 hours.

### 1.2 Meteorological Context of Cyclone Yvette

In mid-May 2014, Mediterranean Cyclone Yvette generated severe wind conditions in the Carpathian Basin, with sustained N-NNW gusts reaching 100–130 km/h. This wind direction is perpendicular to the longitudinal axis of the lake, simultaneously exciting both longitudinal and transverse water level fluctuations. Due to these extreme characteristics of Cyclone Yvette, this specific time period was selected to capture and verify the lake's documented oscillation periods.

## 2 Data and Preprocessing

### 2.1 Hydrodynamic Roles of the Gauging Stations

For validating the frequency response, I chose the dataset with 5 minute measuring differences, because its finesse is better for shorter time periods. Therefore the three measuring stations, where Keszthely (westernmost tip), Siófok (middle of southern bank, eastern basin) and Balatonfűzfő (northern embayment, eastern basin).

### 2.2 Linear Interpolation and Gap Handling

To satisfy the strict requirement of evenly sampled time series for the Discrete Fourier Transform (DFT), linear interpolation was applied to missing records. For a missing value at time  $t$  between two valid observations  $y_0(t_0)$  and  $y_1(t_1)$ , the value is computed as:

$$y(t) = y_0 + (t - t_0) \cdot \frac{y_1 - y_0}{t_1 - t_0} \quad (2)$$

A strict threshold was implemented to prevent artifact creation: gaps exceeding 12 consecutive points (60 minutes) were excluded from interpolation.

### 2.3 Stationarization and Detrending

To remove the global low-frequency drift and stabilize the Fast Fourier Transform (FFT) calculation, a global linear detrending was applied to the entire time series. A first-order polynomial (linear trend)  $y_{\text{trend}}(t)$  was fitted to the data using the method of least squares:

$$y_{\text{trend}}(t) = a \cdot t + b \quad (3)$$

where  $a$  is the slope of the trend and  $b$  is the intercept. The detrended water level anomaly used for the spectral analysis is then obtained by subtracting this linear trend from the original signal:

$$y_{\text{detrended}}(t) = y(t) - y_{\text{trend}}(t) \quad (4)$$

Unlike moving average filters, which can introduce artificial low-frequency oscillations and spectral leakage, this global linear approach effectively removes multi-day water level shifts while preserving the amplitude and phase integrity of the natural oscillatory modes.

## 3 Spectral Methodology

### 3.1 Periodogram Estimation and Windowing

The Power Spectral Density (PSD) was estimated using the periodogram method. To mitigate spectral leakage caused by finite window slicing, a Hann window function  $w(n)$  was applied:

$$w(n) = 0.5 \cdot \left[ 1 - \cos \left( \frac{2\pi n}{N-1} \right) \right], \quad 0 \leq n \leq N-1 \quad (5)$$

### 3.2 Zero-Padding

To enhance the visual and mathematical resolution of the discrete frequency bins in the long-period domain, the dataset was padded with zeros to eight times its original length ( $N_{\text{FFT}} = 8 \cdot N$ ). This mathematical interpolation yields smooth, interpretable spectral curves.

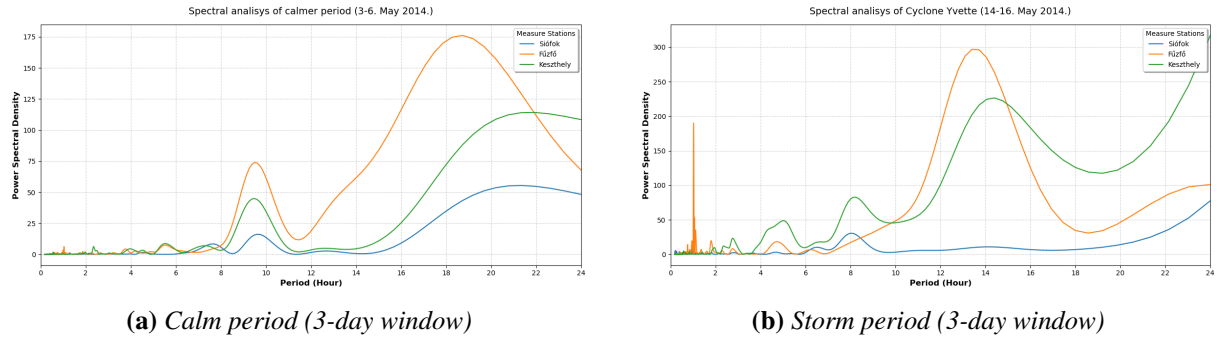
## 4 Results and Hydrodynamic Interpretation

### 4.1 Influence of the Analysis Window Length: A Critical Comparison of 3-Day and 10-Day Windows

A critical methodological finding emerged regarding the choice of the analysis window length ( $N$ ). Initially, a narrow 3-day window (May 14–16) directly covering the peak of the cyclone was analyzed, but the results exhibited severe spectral and mathematical distortions:

- **Storm Period:** The spectra failed to isolate sharp peaks in the expected fundamental frequency band, instead showing broad, shifted energy and rising sharply toward the 24-hour limit. This 24-hour distortion is a mathematical artifact. Linear detrending removes straight-line shifts but leaves behind the non-linear, multi-day "bump" of the storm surge. The Fast Fourier Transform (FFT) algorithm misinterprets this residual curvature as massive, ultra-low-frequency energy, suppressing finer details.
- **Calm Period (May 3–6):** The 3-day window generated broad, smeared peaks (e.g., an 18–19 hour band at Balatonfűzfő). Due to poor frequency resolution (spectral leakage) inherent to short timeframes, the FFT could not separate the lake's true free oscillations from localized, daily diurnal wind cycles, merging them into artificial macro-peaks.

To eliminate these distortions, the analysis windows were uniformly extended to 10 days (Storm: May 10–20; Calm: May 3–12). The longer duration drastically improved frequency resolution, decomposing the smeared energy blocks into distinct, physically meaningful peaks. Importantly, extending the window introduces "energy dilution": because the extreme kinetic energy of the 2–3 day cyclone is averaged over several calmer days, the peak Power Spectral Density (PSD) values in the 10-day spectrum are naturally lower than in the 3-day analysis.



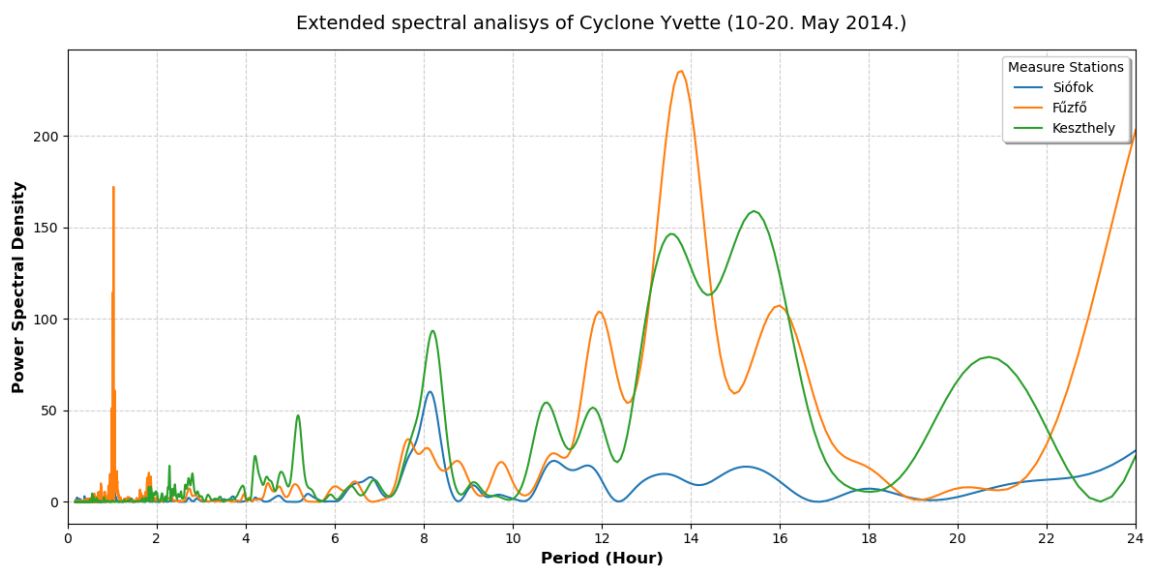
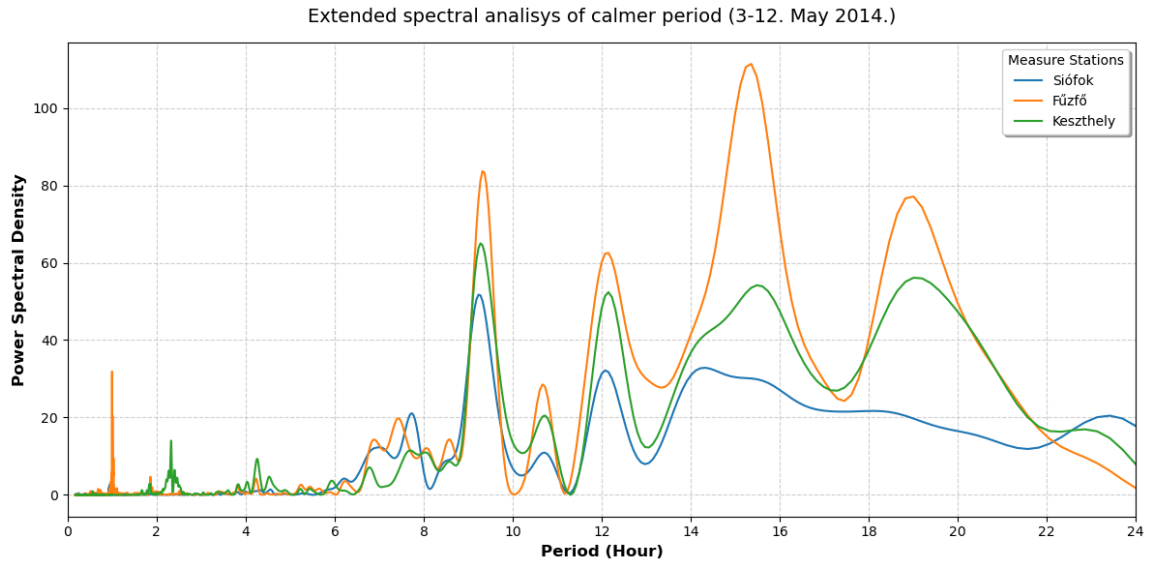
**Figure 1:** Comparison of spectral analysis using a short 3-day window. Both periods exhibit severe mathematical artifacts in the low-frequency domain.

## 4.2 Storm Period Dynamics (May 10–20)

The spectral analysis results are summarized in Table 1.

**Table 1:** Identified spectral peaks and hydrodynamic interpretations during Cyclone Yvette.

Station	Period (Hours)	Max PSD	Hydrodynamic Phenomenon
Keszthely	13.5 – 15.5	~270	Longitudinal fundamental forced mode
Keszthely	8.2	~90	Secondary higher harmonic ( $m = 2$ )
Siófok	11.5 – 14.0	< 15	Proximity to the nodal line (flat response)
Siófok	8.2	~85	Higher harmonic displacement
Balatonfűzfő	13.5 – 14.0	~300	Eastern end of the longitudinal mode
Balatonfűzfő	1.0	~180	Local transverse seiche mode



**Figure 2:** Comparison of robust spectral analysis using the optimized 10-day window. The extended timeline successfully resolves low-frequency storm surge distortions, clarifying the  $\sim 14$ -hour longitudinal resonance and the sharp 1-hour transverse seiche at Balatonfűzfő.

### 4.3 Longitudinal Forced and Free Oscillations (11–16 Hours)

The dominant feature of the 10-day storm spectrum is a complex, multi-peak energy maximum between 11 and 16 hours. Instead of a single sharp peak typical of free oscillations, a broad structure appears, shifting away from the theoretical 11.5–12 hour natural frequency to peak around 13.5–15.5 hours at Keszthely and Balatonfűzfő. This is a classic hydrodynamic response to forced vibration. The continuous, asymmetrical wind stress of Cyclone Yvette prevented the water body from oscillating freely, resulting in an interference pattern between the lake's natural resonance and the atmospheric forcing period.

### 4.4 Behavior of Siófok and Verification of Nodal Line Theory

The spectral response of Siófok strongly validates the theoretical nodal line of the uninodal seiche. While the spectra of the antinodes at Keszthely and Balatonfűzfő show massive energy peaks in the 13–15 hour band (reaching  $\sim 270$  and  $\sim 300$  PSD, respectively), the Siófok curve remains nearly flat. Since vertical displacement at a pure node is theoretically zero, the empirical spectral muteness of this station proves its location on the primary longitudinal nodal line.

### 4.5 The 1-Hour Transverse Seiche at Balatonfűzfő

The Balatonfűzfő spectrum reveals a unique local anomaly: a sharp, isolated peak at a 1.0-hour period that is entirely absent at Keszthely and Siófok. The N-NNW wind direction of Cyclone Yvette blew perpendicularly across the eastern basin, pushing water directly into the Balatonfűzfő embayment. This localized perpendicular forcing induced a distinct transverse seiche, oscillating between the opposite shores with a precise 1-hour rhythm.

### 4.6 The 8.2-Hour Binodal Higher Harmonic

Benefiting from the fine resolution of the 10-day window, a secondary peak was clearly identified at 8.2 hours at both Keszthely and Siófok. This corresponds to the binodal ( $m = 2$ ) longitudinal higher harmonic mode. In a binodal oscillation, the nodal lines shift compared to the uninodal mode, placing Siófok outside the calm zone. Its simultaneous energetic displacement alongside Keszthely confirms the excitation of multi-modal resonance.

### 4.7 Reference Spectrum of the Calm Period (May 3–12)

The 10-day calm window highlights critical hydrodynamic contrasts when external forcing is removed:

- **Energy Levels:** While storm-period peaks exceeded 270 PSD, the background energy during the calm period remained largely below 100 PSD.
- **Reduction of Transverse Resonance:** The sharp 1-hour peak at Balatonfűzfő did not completely disappear, but its energy drastically dropped to approximately 30 PSD. This indicates that while the local transverse seiche is continuously maintained by normal background wind conditions, its extreme amplification is strictly a storm-driven event.

- **Complex Background Oscillation:** The forced 14-hour band is replaced by distinct, lower-energy peaks around 9, 12, and 15.5 hours. This proves that without severe storms, daily thermal wind cycles and minor atmospheric pressure fluctuations maintain a continuous, multi-component background oscillation.

## 5 Conclusions

This study successfully verified the multi-modal seiche characteristics of Lake Balaton using high-resolution 5-minute water level data. Methodologically, an extended 10-day analysis window combined with global linear detrending was proven essential to eliminate low-frequency mathematical artifacts and isolate longitudinal and transverse oscillations cleanly. The near-zero spectral response at Siófok in the 13–15 hour band empirically confirms its position on the primary nodal line of the uninodal seiche. Furthermore, the analysis demonstrated that while the 1-hour transverse resonance in the eastern basin is a continuous background phenomenon, its extreme amplification is strictly a storm-driven event triggered by perpendicular wind forcing.

## References

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