Alternative definition for the ribbon graph polynomial

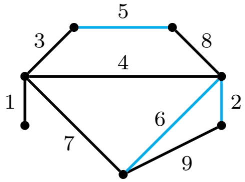
Páhán Anita Dalma

Math project 2

Supervisor: Tóthmérész Lilla

Tutte polynomial

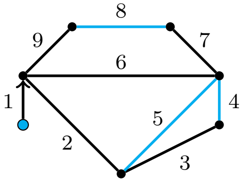
- 2-variable graph polynomial
- several definitions
  - deletion-contraction
  - sum over spanning trees



$$T(G;x,y) = \sum_{T \text{ spanning tree}} x^{IA(T)} y^{EA(T)}$$

Bernardi's definition

- tour of a spanning tree
  - starting point and edge
  - sequence:
    1 2 3 4 3 5 2 6 5 4 7 8 7 6 9 8 9 1
  - interlacing edges



$$T(G;x,y) = \sum_{T \text{ spanning tree}} x^{IA'(T)} y^{EA'(T)}$$

Embedded graphs

- ribbon structure
- Bollobás and Riordan: 4-variable polynomial
  - specialization: ribbon graph polynomial

$$T(\mathbb{G},x,y) = f(e)T(\mathbb{G}\backslash e) + g(e)T(\mathbb{G}/e)$$

$$f(e) = \begin{cases} x-1 & \text{if } e^* \text{ is a loop in } \mathbb{G}^* \\ 1 & \text{if } e^* \text{ is not a loop in } \mathbb{G}^* \end{cases} \quad (1)$$

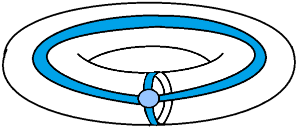
$$g(e) = \begin{cases} y-1 & \text{if } e \text{ is a loop in } \mathbb{G} \\ 1 & \text{if } e \text{ is not a loop in } \mathbb{G} \end{cases} \quad (2)$$

$\mathbb{G}^*$: dual graph, e^*: dual of edge

Quasi-tree

Definition

We call a subset of edges a quasi-tree if the corresponding ribbon graph has exactly one boundary component. A spanning quasi-tree is a quasi- tree which contains all vertices.



- ribbon graph polynomial as sum over spanning quasi-trees

$$T(\mathbb{G},x,y)=\sqrt{x-1}^{\,e(\mathbb{G})-b(\mathbb{G})+1}\cdot\sqrt{y-1}^{\,1-v(\mathbb{G})}\sum_{\mathbb{T}}\sqrt{\frac{y-1}{x-1}}^{\,e(\mathbb{T})}x^{ILO(\mathbb{T})}y^{ELO(\mathbb{T})}$$

Bernardi's definition

Theorem

The previous equation remains valid if we use the embedding-activities defined by Bernardi instead of the original activity definition.

Proposition

An edge is contained in every/none of the spanning quasi-trees if and only if for an arbitrary spanning quasi-tree there is no interlacing edge with it.

Use of AI

- In my work I used AI for language checking.

Thank you for your attention!