Vertex matroid families

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Matroid families

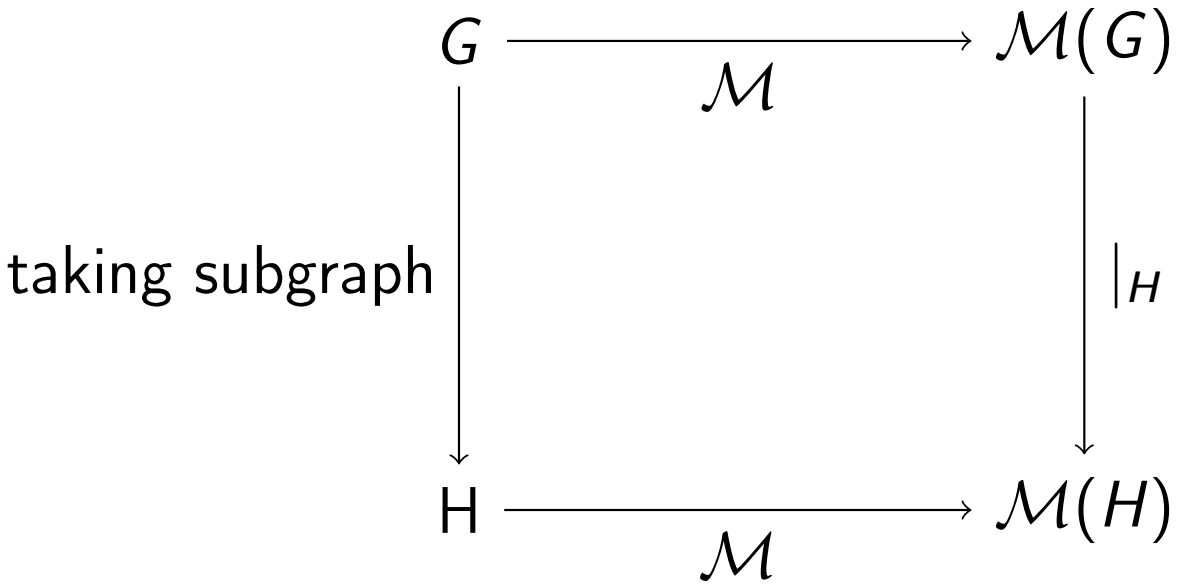
Definition

*A **graph matroid family** $\mathcal{M}$ is a family of matroids, defined on the edge set of each (finite, simple) graph in a way that is*

- *well-defined: every graph isomorphism $\varphi : V(G) \to V(H)$ induces an isomorphism between $\mathcal{M}(G)$ and $\mathcal{M}(H)$.*
- *compatible: for all subgraph H of G, $\mathcal{M}(H)$ is a restriction of $\mathcal{M}(G)$.*

Graph matroid families

For a given graph $G = (V, E)$ and a graph matroid family $\mathcal{M}$.



Matroid families

Some known graph matroid families:

- graphic matroids
- bicircular matroids
- even cycle matroids
- abstract rigidity matroids
- (k, l)-count matroids

Vertex matroid families

Definition

*A **vertex matroid family** $\mathcal{M}$ is a family of matroids, defined on the vertex set of each (finite, simple) graph in a way that is*

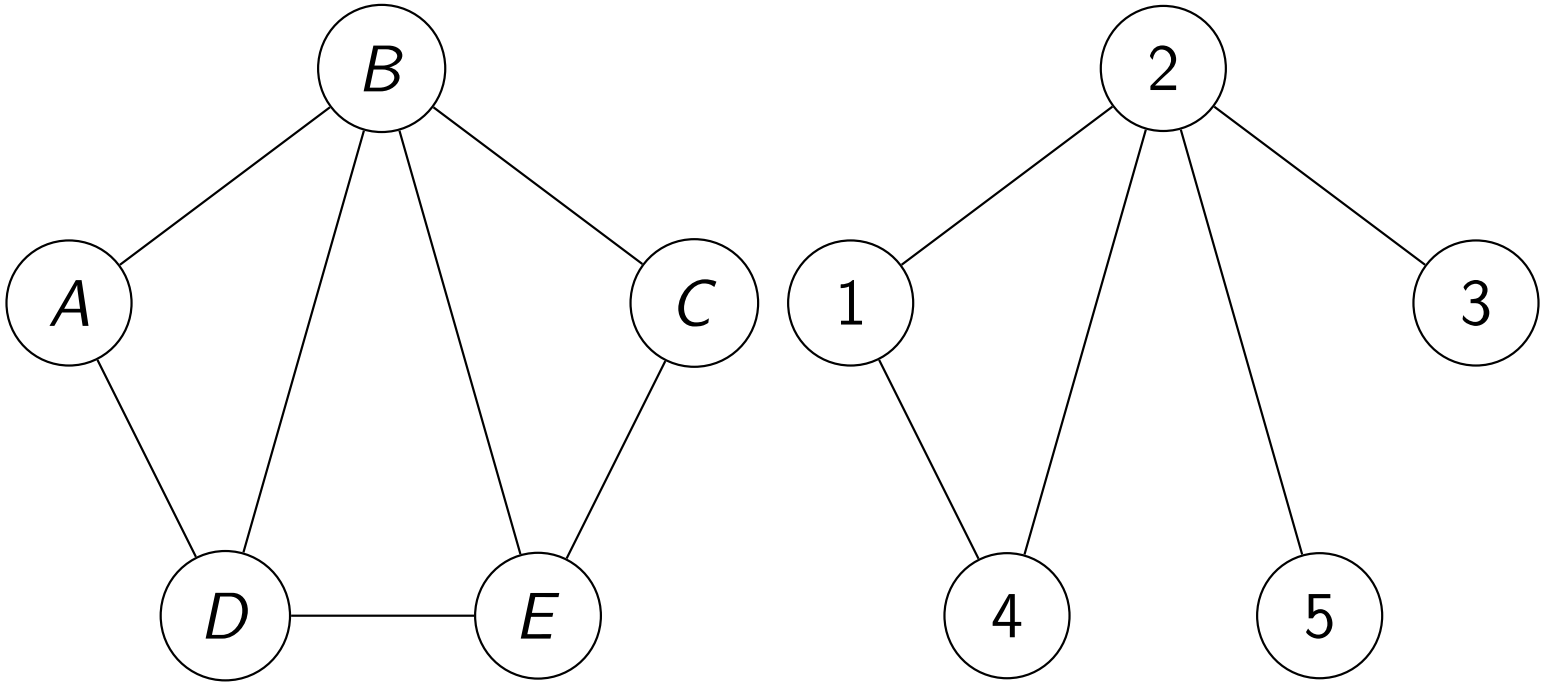
- *well defined: every graph isomorphism $\varphi : V(G) \to V(H)$ induces an isomorphism between $\mathcal{M}(G)$ and $\mathcal{M}(H)$.*
- *compatible: for all induced subgraph H of G, $\mathcal{M}(H)$ is a restriction of $\mathcal{M}(G)$.*

Main Theorem

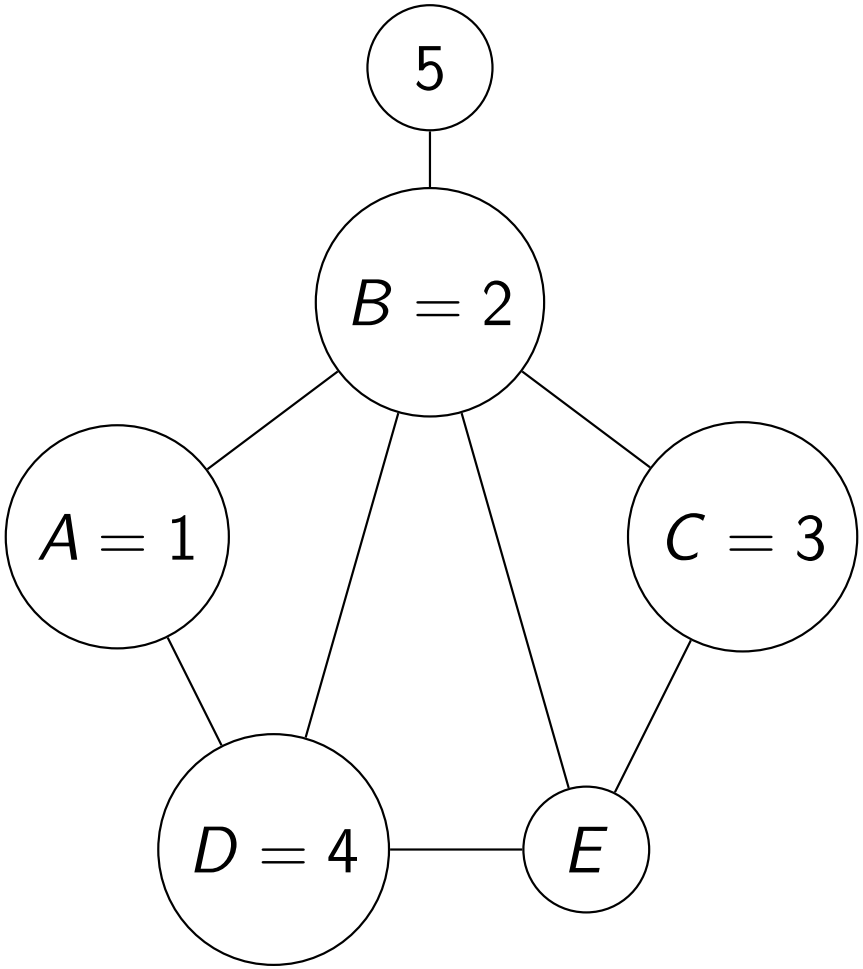
Theorem

For all vertex matroid family $\mathcal{M}$ either all matroids are free, or there exists an integer k such that for all finite simple graphs G, $\mathcal{M}(G)$ is the uniform matroid on $|V|$ elements with rank $\min\{|V|, k\}$.

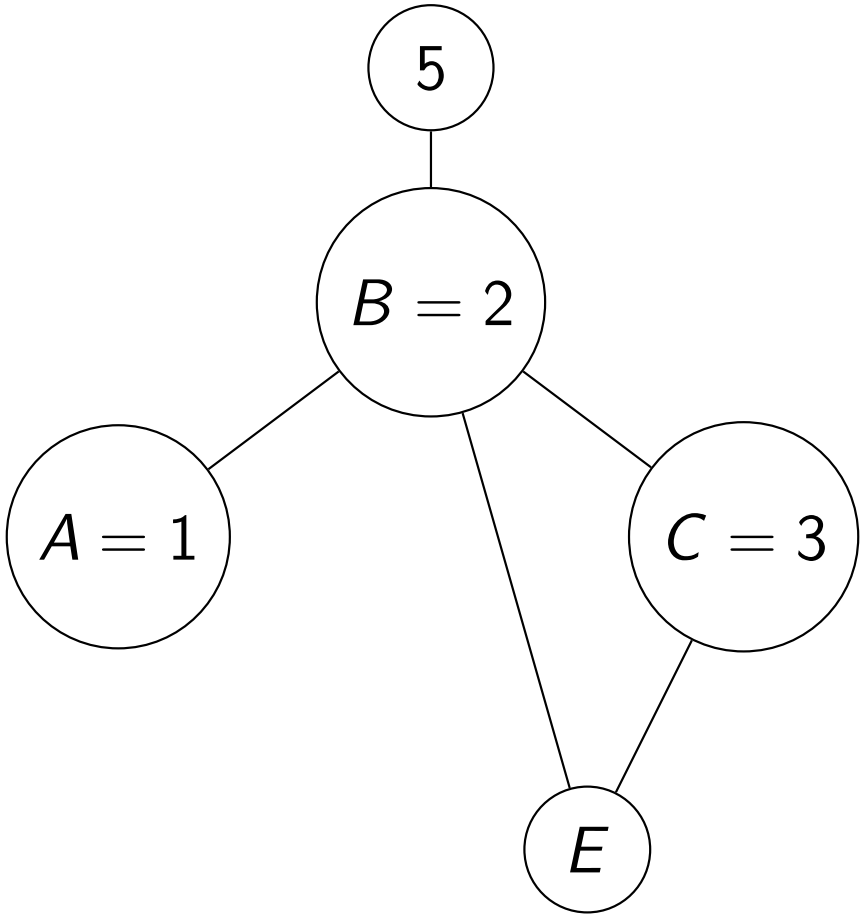
Gluing operation



Gluing operation



Use of gluing operation



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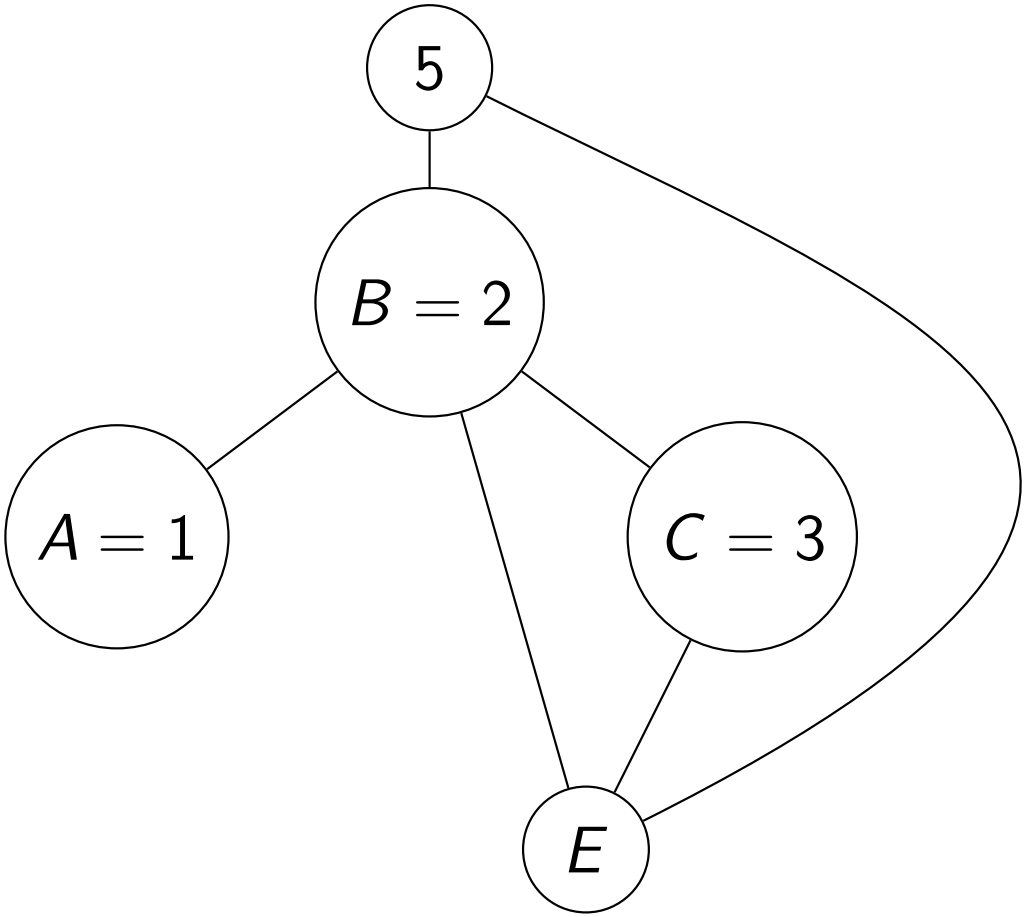

Thank you!