Power Spectral Analysis of Seiches in Lake Fertő

Láng Kristóf
Supervisor: Dr. Krámer Tamás

Eötvös Loránd University, Faculty of Science

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1 Introduction

Basin-scale oscillations, also known as seiches, are standing waves in enclosed or partially enclosed bodies of water. In lakes, they are formed when the effect of external forces (most often wind and changes in atmospheric pressure) cause the water in the basin to oscillate for longer periods of time. The ultimate goal of this project is to identify and characterize seiches in Lake Fertő using time series data from multiple shoreline gauge stations. By applying a combination of spectral, cross-spectral, and statistical methods, the project seeks to determine which oscillatory modes are detectable in observations, how their properties vary spatially along the shoreline, and how they relate to the physical characteristics of the lake. A particular challenge is that Lake Fertő is considerably smaller and much shallower than the large lakes typically analysed in the seiche literature, with considerably dense vegetation, implying potentially stronger damping and less persistent oscillations.

My main objective for the first part of this project work was to acquire a rigorous understanding of the theoretical background of the studied research ([Man20]), while also conducting a few initial analyses based on this background.

2 Theoretical Background

As per the referenced study [Man20], at basin scale, lake seiches can be approximated as long, shallow-water waves. In their simplest form, the dynamics of a single basin mode may be reduced to that of a forced, linearly damped harmonic oscillator. The governing equation of this "first basin mode" can be written as

$$\frac{d^2A}{dt^2} + 2r\frac{dA}{dt} + \omega_n^2 A = f(t), \tag{1}$$

where A is the modal amplitude, r is the damping coefficient, ω_n is the natural seiche frequency, and $f(t)$ represents the external forces. Damping r is to be calculated via the quality factor Q, using the relation $r = \frac{\omega_{peak}}{2Q}$. Q is the factor that measures the ratio of stored energy to energy lost per oscillation cycle, and is often calculated as the ratio of the resonance frequency divided by the width of the frequency region falling within 3db of the response at resonance frequency. A relatively high Q indicates that the oscillations persist for longer periods of time, while a low Q is the sign of a more damped system where the oscillations decay quickly. In the case of Lake Fertő, the initial expectation is a low Q, as the average water level is shallow and vegetation is high.

2.1 Power Spectral Analysis

Power spectral density (PSD) analysis quantifies the distribution of variance in a time series as a function of frequency. As per [Sem11], the PSD may be estimated from the squared magnitude of its Fourier transform:

$$PS(\omega) = |Y(\omega)|^2 \tag{2}$$

where

$$Y(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}dt \tag{3}$$

if f(t) denotes the sampled data. Since water level measurements are on a discrete time scale (15 minutes for the data used in this project), in practice an efficient implementation of the discrete Fourier transform, the fast Fourier transform can be used.

Deriving the power spectral density of the sampled data is a powerful tool here as in an ideal case the frequency of the first basin mode can be easily read from the resulting periodograms, as it will form a peak in resonance at all the measuring points. As multiple basin modes exist, the resonance peak at the lowest frequency should be identified as the first basin mode in the relevant frequency range. A decent estimate fot the longest natural period T that can aid the identification process can be computed using Merian's equation:

$$T = \frac{2L}{\sqrt{gh}} \tag{4}$$

where L and h are the length and average depth of the body of water, and g is the acceleration of gravity.

2.2 Cross-spectra and Coherence

As basin-level oscillations are inherently spatially coherent, periodograms from individual measuring stations are unable to fully describe them. Cross-spectral analysis can provide a measure of coherence between pairs of measuring stations on the frequency domain. Taking two time series $y_1(t)$ and $y_2(t)$, their cross-spectra can be calculated as

$$\gamma_{12}^2(\omega) = \frac{|S_{12}(\omega)|^2}{S_{11}(\omega)S_{22}(\omega)} \tag{5}$$

where S_{11} is the one-sided spectrum, S_{12} is the one-sided cross-spectrum, and is calculated via the Fourier transform $Y(\omega)$ of time series $y(t)$ as

$$S_{12}(\omega) = \frac{2}{N\Delta t}[Y_1^*(\omega)Y_2(\omega)], \tag{6}$$

N denoting the number of data points. Coherence can be used to verify the existance of a basin-wide mode. In the referenced study [Man20] it is calculated for all stations against a fixed reference station, and all coherence figures show a peak at the theoreticised first basin mode frequency.

2.3 EOF

Empirical Orthogonal Functions (also known as PCA or SVD) can be used to decompose the variance of time series data into modes with relative contributions from each station. When ordered by the measure of explained variance, the coefficients of the first EOF are often similar at all stations (representing seasonal or longer term variability of the entire lake level), while the coefficients of the second EOF are often ordered as the stations themselves are ordered along the dominant axis of the lake, with negative coefficients on one side, and positive coefficients on the other. This behavior suggests that the second EOF represents the basin-wide seiche mode.

3 Initial Results

This project work uses measurements from 8 measuring stations along the shoreline of lake Fertő, with data points from between 2009 and 2015, sampled at 15 minute intervals. The analysis was carried out in MATLAB. For the first part of the analysis, the measurements of station Fertőrákos were omitted, as they contained high levels of missing values. Some figures of the initial analysis are presented below.

Power spectra were computed using the direct periodogram approach and expressed in cycles per day. Spectral estimates were stabilized through frequency-domain band-averaging and examined for the full record, as well as on a year-by-year and seasonal basis. Unlike the referenced study, the power spectra did not show clear resonance peaks and remained quite noisy even after high levels of band-averaging. The only peak identifiable at all stations was around the 24 cpd frequency, suggesting hourly oscillations, but still was not clear enough to be conclusive.

EOF analysis revealed a dominant first mode explaining the majority of variance, with nearly uniform loadings across all stations. This mode is interpreted as basin-wide, low-frequency variability associated with seasonal and/or longer-term water-level changes. The second EOF exhibited a systematic sign change between the northern and southern shores, with intermediate stations ordered approximately monotonically in between, consistent with the spatial structure expected for a longitudinal basin-scale oscillation (and the results of the referenced research).

Despite this physically plausible spatial pattern, the associated temporal coefficients and station-level PSDs did not exhibit a sharp, persistent spectral peak. This suggests strong damping and intermittent excitation of the oscillation, in agreement with expectations for a shallow, vegetation-rich lake such as Fertő.

As for the future of the project work, cross-spectral analysis will be carried out to examine the between-station coherence of data, and additional data preparation methods will be explored to potentially reduce noise levels. Additionally, subsequent analysis methods of the referenced study will be explored for further insights.

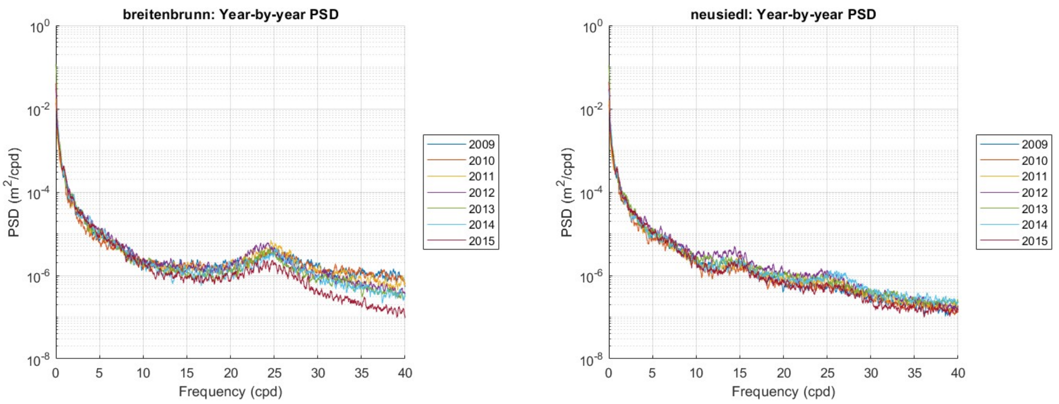

Figure 1: PSD of two stations, separating yearly data

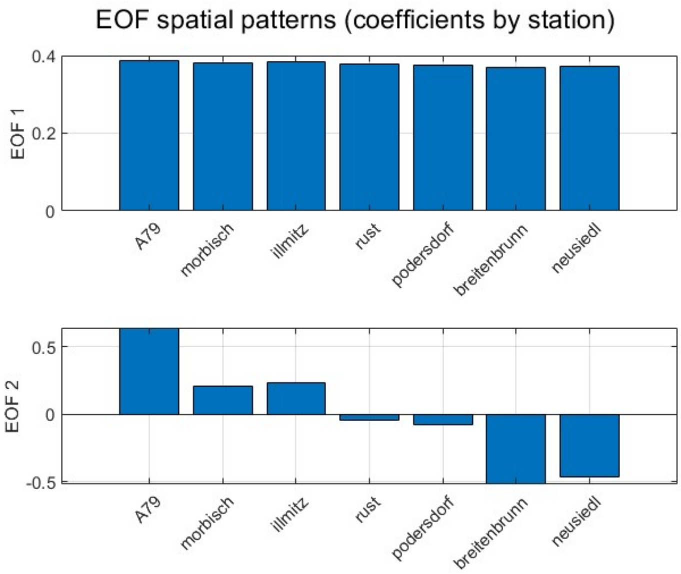

Figure 2: Coefficients of the first two EOF by station.

References

[Man20] Maqsood Mansur. Observation and prediction of seiches in lake superior. Master's thesis, University of Minnesota, 2020. Accessed: 2025-10-28.

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