Power Spectral Analysis of Seiches in Lake Fertő

Láng Kristóf

Project Work I.
Supervisor: Dr. Krámer Tamás

08-01-2025

Agenda

Motivation

Seiches

- Seiches are standing waves
- Formed by external forces (wind, atmospheric pressure)
- The oscillation of water persists for longer periods of time
- Seiche periods can only be roughly estimated from theory alone

Lake Fertő

- Small basin, shallow water, high level of vegetation
- Strong damping, less persistent oscillations expected

Forced Damped Harmonic Oscillator

Overarching equation: **Forced, damped harmonic oscillator**
The evolution equation of amplitude A can be given by

$$\frac{d^2A}{dt^2} + 2r\frac{dA}{dt} + \omega_n^2 A = F \tag{1}$$

- r is the damping coefficient
- ω_n is the natural seiche frequency
- **F** is the external forcing

r and ω_n are to be estimated using power spectral analysis
r can be estimated through its relation to the Q factor:

$$r = \frac{\omega_{peak}}{2Q} \tag{2}$$

$$Q = \frac{\omega_{peak}}{\Delta\omega_{-3dB}} \tag{3}$$

Power Spectral Analysis

To determine ω_{peak}, we use power spectral analysis.
We convert the time-dependent measurements to the frequency domain

$$PS(\omega) = |Y(\omega)|^2 \tag{4}$$

here, $Y(\omega)$ denotes the Fourier transform:

$$Y(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \tag{5}$$

while $f(t)$ is the lake level data from measurements.
Practice:

- Data is on a discrete scale
- Fast Fourier Transform is used
- Hope: an easily detectable peak at the frequency where energy is the strongest

Empirical Orthogonal Functions

- More generally called Principle Component Analysis
- Used to capture the dimensions in the data that make up most of the variance
- Here: used to deconstruct lake level variability into modes that explain a large part of this variability
- Primary goal: confirm the existence of a basin wide seiche

Data Preparation

- Data used from 8 stations around Lake Fertő
- Water level data taken in 15 minute intervals between 2009 and 2015
- Fertőrákos initially omitted
- Before analysis the data was detrended, tide removal is deemed unnecessary
- Prior to EOF analysis, the mean water level at each station was substracted
- Different levels of band-averaging used to smooth out data

EOF: Results

An EOF analysis was carried out for all the stations
Expectation:

- First principal component determining lake-wide water level variations
- Second PC characterising the primary seiche (first basin mode)

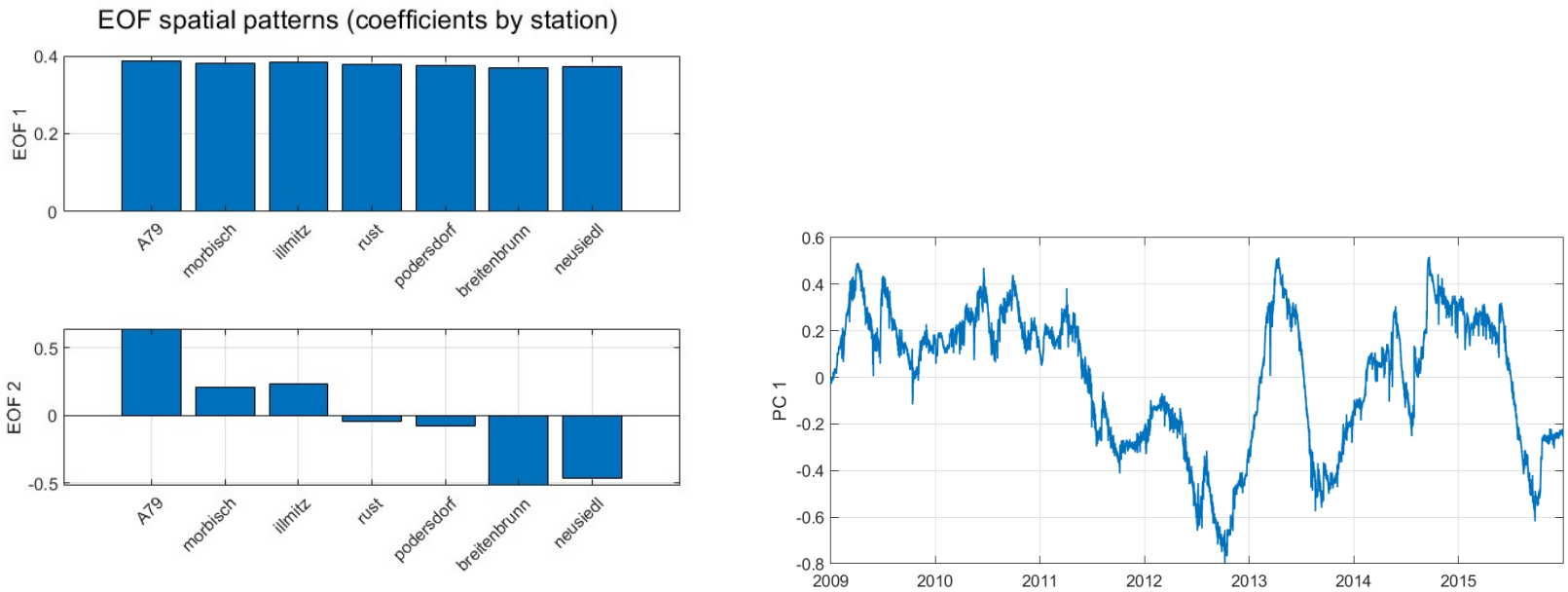

Figure 1: Coefficients of the first two EOF by station, and time series of the first Principal Component

- The first EOF explains most of the variance

PSD: Results

PSD calculated for all stations across the entire time period, and yearly as well
Band-averaging to negate noise
Expectation: Clear peak at resonance frequency

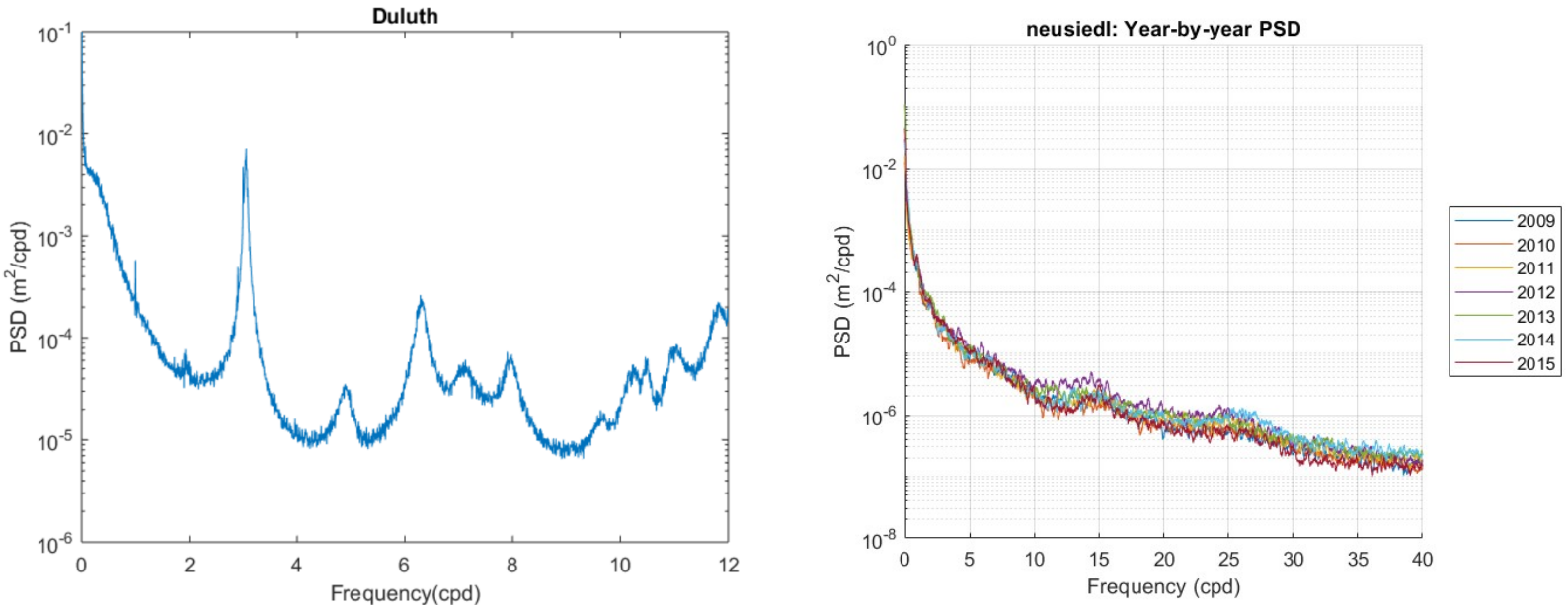

Figure 2: PSD of the reference study for Lake Superior vs. PSD for Lake Fertő at station Neusiedl am See

Future Work

- Explore different data preparation methods for clearer results
- Perform Cross-Spectral Analysis, determine coherence
- Estimate damping and Q factor
- Explore additional methods used by the reference study

Thank you for your attention!

Usage of Artificial Intelligence

During my project work, I used ChatGPT to help with some unclear parts of code implementation.

References:

Maqsood Mansur. Observation and prediction of seiches in lake superior. Master's thesis, University of Minnesota, 2020.