Sandpile group actions

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Math project 1

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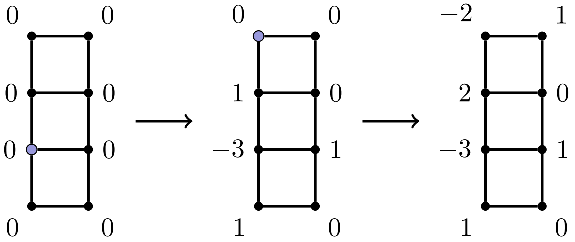
Introduction

- connected, undirected graph G
- chip configuration: $c \in \mathbb{Z}^V$
    - additive group $\mathbb{Z}_0^V$
    - chip-firing game



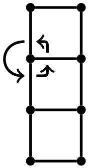
Introduction

- connected, undirected graph G
- chip configuration: $c \in \mathbb{Z}^V$
  - additive group $\mathbb{Z}_0^V$
  - chip-firing game
  - equivalence relation $\sim_G$
- sandpile group $S(G)$
  - number of elements $=$ number of spanning trees in G

Sandpile group

Rotor-routing

- ribbon structure
  - plane graph



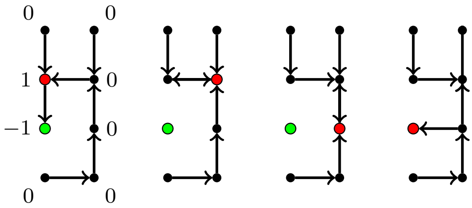
Proposition

For any graph G with ribbon structure exists an orientable surface in which it can be embedded.

Sandpile group

Rotor-routing

- ribbon structure
- root
- spanning tree, orientation of edges



Consistency

Definition

A sandpile torsor algorithm is consistent if for every plane ribbon graph, every tree T and every $s, t \in V(G)$ (such that $st \in E(G)$) the following properties hold

- *For any $e \in E(G)$ where $e \neq st$ if $e \in T$ and $e \in \alpha_G(s-t, T)$:*
$$\alpha_{G/e}(s-t, T \setminus e) = \alpha_G(s-t, T) \setminus e$$
- *For any $e \in E(G)$ if $e \notin T$ and $e \notin \alpha_G(s-t, T)$:*
$$\alpha_{G \setminus e}(s-t, T) = \alpha_G(s-t, T)$$
- *For any $e \in E(G)$ such that $e \neq st$, if there exists cut vertex x, and st and e are on different sides of x, then*
$$e \in T \iff e \in \alpha_G(s-t, T)$$

Consistency

Plane graph

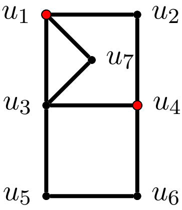
Theorem

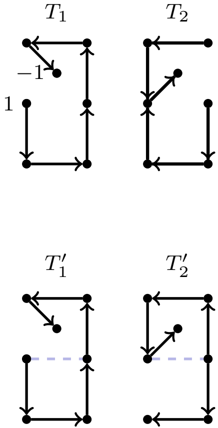
The rotor-routing algorithm is consistent.

Non-plane graph

- depends on the root
- a cycle does not divide the surface into two parts

Example





Use of AI

- In my work I used AI for language checking.

Thank you for your attention!