# Positional Games

## Directed studies I presentation

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# Introduction

- reading Positional games book from authors Dan Hefetz, Michael Krivelevich, Miloš Stojaković and Tibor Szabó
- Motivation: Miloš Stojaković minicourse
- read the first three chapters and solved most exercises

# Definition

A positional game is played on a set $X$ (usually finite), the board of the game and the two players take turns occupying previously unoccupied elements of the board. In the most general version, there are two parameters $p, q$, the first player takes $p$ unoccupied elements of $X$ and the second player takes $q$ unoccupied elements in his turn. There is a given set of subsets, which the players focus on. A "Maker" player tries to claim a whole winning set, "Breaker" player tries to prevent the opposing "Maker" player from claiming a winning set. An "Avoider" player tries to not claim a whole losing set and an "Enforcer" tries to prevent the opposing "Avoider" player. The four main types of studied positional games are Maker-Maker (The first player who claims a whole winning set wins), Maker-Breaker, Avoider-Enforcer and Avoider-Avoider. The game ends in a draw if none of the Makers succeed or both of the Avoiders succeed.

# Examples of Positional Games

Classic examples include:

- Tic-Tac-Toe and its generalizations ($n^d$). Tic-Tac-Toe is a draw by pairing strategy for boards $\geq 5 \times 5$.
- Hex, Connectivity game, Sim.
- Hamiltonicity and row-column games.

# Strong Games (Maker-Maker)

Both players try to claim a winning set first.

- Second player cannot win due to strategy stealing. Strategy stealing: If second has winning strategy, first plays random then follows it
- Ramsey-type results in many cases imply no draws
- Open: explicit strategies

# Maker-Breaker Games

Maker (first) tries to claim a winning set; Breaker blocks.
The Erdős-Selfridge criterion states that for a set of winning sets $\mathcal{F}$, then if $\sum_{A \in \mathcal{F}} 2^{-|A|} < \frac{1}{2}$, then the game is Breaker's win.
Idea: Breaker decreases "danger" of the hypergraph
Applications: 2-colorings of graphs, clique game, $n$-in-a-row game

# Biased Games

Maker takes $p$ elements, Breaker $q$.

Generalized version of the Erdős-Selfridge theorem: If $\sum_{A \in \mathcal{F}} 2^{-|A|/p} < \frac{1}{1+q}$, then Breaker (as Second Player) has a winning stretegy for the game. winning threshold is connected in many cases to random graphs, for example Connectivity

# What's next?

- Questions
- You: Conference
- Me: Bridge competition

Thank you for your attention!