Spectral analisys of Lake Balaton seiche

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1 Introduction

Lake water levels exhibit not only long-term seasonal variations but also characteristic oscillations on shorter time scales. One important example is the seiche, a damped oscillation of the water surface around its equilibrium position, triggered by wind forcing and atmospheric pressure variations. From a physical point of view, seiches can be interpreted as shallow-water waves whose natural frequencies depend on the geometry of the basin, the water depth, and the shape of the shoreline.

Due to its shallow depth and elongated shape, Lake Balaton is particularly susceptible to the formation of seiches. Even moderate wind events can induce noticeable water level fluctuations, which appear with different amplitudes and phases at various locations along the shore. Beyond their theoretical interest, seiches have practical relevance, as they influence harbor dynamics, shoreline flooding risks, and ecological processes in nearshore areas.

Water level time series from Lake Balaton are analyzed using spectral analysis techniques, based on the methodology applied to Lake Superior by Mansur (2020) [2]. Fourier-based power spectra are used to identify the dominant oscillation periods, while cross-spectral and coherence analyses help reveal phase relationships between different measurement stations. In addition, wavelet analysis is applied to investigate the temporal evolution and damping of seiche events. The results are interpreted using simple physical considerations, linking the observed spectral characteristics to the geometric and hydrodynamic properties of Lake Balaton.

2 Methodology and Mathematical Background

This study is based on the analysis of water level time series recorded at several shoreline stations of Lake Balaton. The data are assumed to be evenly sampled in time after basic preprocessing steps, including the removal of missing values and linear detrending. The applied methods aim to identify characteristic oscillation periods, spatial relationships between stations, and the temporal evolution of seiche events.

2.1 Power Spectral Analisys

Power spectral analysis is employed to describe the distribution of variance in water level time series as a function of frequency. Prior to the analysis, the data are detrended in order to suppress low-frequency components not associated with seiche oscillations. The power spectral density (PSD) is defined as the magnitude squared of the Fourier transform of the signal [1],

$$P(f) = |X(f)|^2,$$

where $X(f)$ denotes the Fourier transform of the time series $x(t)$.

The resulting spectral representation can be used to identify frequency ranges associated with seiche oscillations. Based on the spectral characteristics of these oscillations, damping properties can be quantified through the estimation of a quality factor Q, which characterizes the ratio of stored to dissipated energy in the system.

The power spectra will be used to identify the seiche frequency in Lake Balaton. The dominant peak in the spectra corresponds to the natural frequency of the seiche. This peak frequency, denoted as f_0, represents the fundamental mode of the transverse oscillation of the lake.

2.2 Cross-Spectral Analysis

While power spectra provide information about the energy distribution at individual locations, they do not reveal the spatial structure of the oscillations. To address this, cross-spectral analysis is performed between pairs of water level stations [3]. This method allows for the determination of the coherence and phase relationship between two time series, $x(t)$ and $y(t)$.

The cross-spectrum $S_{xy}(f)$ is computed using the Fourier transforms of the two signals. From this, the magnitude-squared coherence, $\gamma_{xy}^2(f)$, is derived as:

$$\gamma_{xy}^2(f) = \frac{|S_{xy}(f)|^2}{S_{xx}(f)S_{yy}(f)}, \tag{1}$$

where $S_{xx}(f)$ and $S_{yy}(f)$ are the power spectral densities of the individual signals. Coherence values range from 0 to 1, where a value close to 1 indicates a strong linear relationship between the signals at a specific frequency. High coherence at the seiche frequency confirms that the oscillations observed at different locations are part of the same basin-wide mode.

2.3 Wavelet Analysis

Fourier analysis assumes that the signal is stationary, providing an average frequency content over the entire observation period. However, seiches are transient phenomena; they are typically generated by impulsive wind or pressure events (storms) and subsequently decay due to friction. To capture this temporal evolution, wavelet analysis is employed.

Unlike the Fourier transform, the continuous wavelet transform (CWT) decomposes the signal into wavelets that are localized in both time and frequency. The transform is defined as the convolution of the water level time series $x(t)$ with a scaled and translated mother wavelet function $\psi(t)$:

$$W(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^* \left(\frac{t-b}{a} \right) dt, \tag{2}$$

where a represents the scale (inverse of frequency), b is the time shift, and the asterisk denotes the complex conjugate.

3 Data Processing and Preliminary Analysis

The quantitative analysis of seiche events relies on high-quality water level records. Observational water-level data were downloaded from the Átfogó Balaton portal of the Hungarian General Directorate of Water Management (OVF). For this study, time series data were examined for the period between 2008 and 2014. The dataset includes records with two different temporal resolutions: 5-minute for Balatonfűzfő, Keszthely and Siófok and 15-minute sampling intervals for all level station around Lake Balaton. The primary objective of the data preparation phase was to produce a homogenized, continuous dataset suitable for spectral analysis.

3.1 Comparison of Temporal Resolutions

A comparative analysis was conducted between the high-frequency (5-minute) and the standard (15-minute) datasets. While the 5-minute resolution offers detailed insight into higher-frequency oscillations, preliminary inspection revealed systematic vertical inconsistencies. Specifically, the 5-minute records occasionally exhibited vertical measurement shifts.

To mitigate these effects, the 5-minute observations were temporally aggregated by computing monthly mean water levels for each measurement station. The Siófok water-level station was selected as a reference station, and relative water-level adjustments were applied to the remaining stations based on the mean inter-station differences. For the 15-minute data, no adjustment was necessary, as the differences between stations in the averages were within the measurement error (1–2 cm)

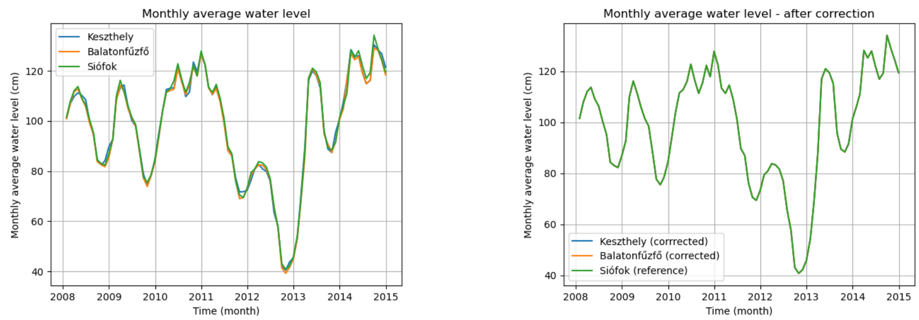

Figure 1: Comparison of 5-minute Keszthely, Balatonfűzfő and Siófok water level records, before and after the correction.

3.2 Gap Analysis and Interpolation

Data continuity is essential for FFT and wavelet-based methods. Therefore, a comprehensive assessment of missing values was performed for the 2008–2014 interval. The distribution of data gaps was mapped to distinguish between short-term instrument failures and longer periods of data unavailability.

Short-term gaps (typically spanning less than 3 hours) were filled using linear interpolation. This method preserves the general trend of the water level changes without introducing significant spectral artifacts in the frequency range of interest. For periods with extensive missing data, the affected time windows were flagged and excluded from the subsequent spectral analysis to strictly maintain statistical reliability.

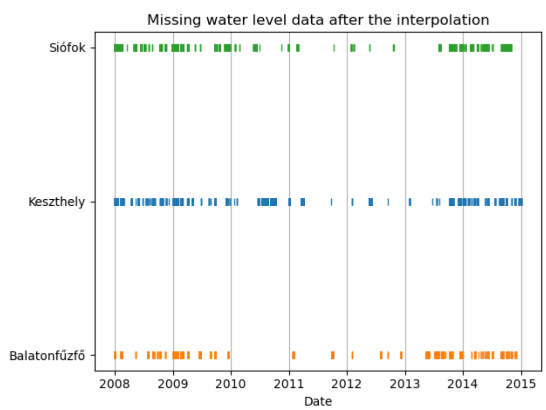

Figure 2: Temporal distribution of missing water-level data after interpolation. The period from February 2011 to March 2013 is clearly the most suitable for investigation.

4 Future work

The upcoming stages of the project will focus on the completion of the theoretical chapters and the implementation of advanced data analyses. Planned tasks include:

- Power spectral analysis to characterize the dominant frequencies in the water-level time series.
- Q-factor estimation to quantify the damping characteristics of the observed oscillatory modes.
- Cross-spectral analysis to investigate the relationships between different measurement stations.
- Wavelet analysis for time-frequency decomposition and detection of transient events.

These steps aim to provide a deeper understanding of the dynamics of the water-level fluctuations and to enhance the overall robustness of the analysis framework.

References

[1] John Semmlow. *Signals and systems for bioengineers: a MATLAB-based introduction.*

[2] M. Mansur. *Observation and Prediction of Seiches in Lake Superior.*

[3] Richard E Thomson and William J Emery. *Data analysis methods in physical oceanography.*