Hoist Scheduling Problem

1 Introduction

This report presents the objectives and progress made in my Student Project on the hoist scheduling problem (HSP). The HSP originates from electroplating lines, where we have to design a fully automated system that is used to cover parts with a coat of metal. The parts are moved by one or more preprogrammed hoists between tanks containing various process specific solutions. The assumptions and constraints involved in the problem vary widely and can lead to highly complex tasks, some of these extensions are mentioned in the final section. As an initial step, the study focused on the single-hoist cyclic HSP, which involved reviewing and analyzing relevant research articles on the topic.

2 Single-hoist CHSP

Let us first consider the problem described in [1]. In this setup parts, stored by carriers, must undergo a sequence of chemical treatment. First the carriers are loaded by a hoist into the system (see Figure 1) from a loading station (tank 0), then transported between the tanks in a given order (1 to n) and then moved back to the loading station. We are given empty-hoist movement times and loaded-hoist movement times as input. The tanks are assumed to be indexed in the order of the soaking process, so the loaded hoist movements are only done between consecutive tanks. The carriers must be immersed into every tank for which minimum and maximum soaking times are given as input. No two carriers may occupy the same tank at the same time. The hoist is programmed to perform the same set of actions over and over. The amount of time between two successive introductions of carriers into the system (from the loading station) is called a *cycle*, which is a *cycle length* long time-span. This is called a simple cycle. It follows that the system must be in the same state after each cycle, meaning the same set of tanks are empty and full initially (tank 0 is always full). For simple cycles it is also true that during a cycle exactly one carrier is dropped into and is removed from each tank, because otherwise the same state could not be attained after the cycle.
The objective is to maximize the throughput of carriers of the system. With the constraint that we want to find a simple cycle, this is equivalent to finding a minimal cycle length.

3 Method

The authors of [1] propose a MIP formulation of the problem, along with procedures of constructing a schedule from the obtained solution. The MIP formulation of the problem can be found in [1]. The most important variables for the interpretation of the solution are $t_i, i \in \{0, 1, \ldots, n\}$, forced to represent the removal times of a carrier for each tank ($t_0 = 0$). This is well defined, because of the single removal during a cycle. The single variable $t_{\max}$ is forced to be the maximum of the t_i's and for each tank the binary variable z_i is defined so that $z_i = 1 \iff t_i = t_{\max}$ (see Type 1 constraints in [1]). The variables y_{ij} are zero-one variables indicating whether a carrier leaves tank i after tank j during the

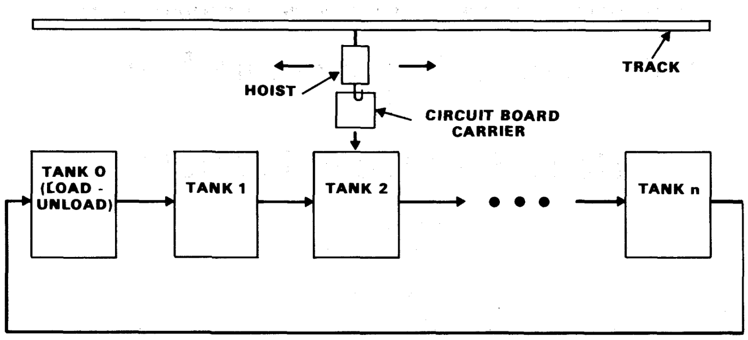

Figure 1: Illustration of the system [1]

cycle. The Type 2 and Type 3 inequalities in [1] ensure that no two carriers occupy the same tank at any time, no two moves are done simultaneously, the hoist has enough time to travel from tanks, where it was used to where it is needed and the soaking times are within the given boundaries. This way every schedule that satisfies all these conditions is a feasible solution of the MIP and every feasible solution of the MIP is a real schedule. If the soaking time boundaries and the travel times are integers then there will be an integer optimal solution as well. Once the MIP is solved, the entry times for each tank can be reconstructed. In fact, for each tank, a feasible time interval can be determined, allowing flexibility in choosing the exact entry time.

4 Implementation and example

For the implementation of the MIP model, the IBM ILOG CPLEX Optimization Studio was employed. The authors of [1] constructed a 12 tanks example, that has become a benchmark example in several subsequent studies. In the original paper the authors used additional constraints to reduce the size of the problem and hence achieve short run times. However this led to the loss of the optimal solution. Thanks to advances in computational power and IP solving techniques, it is now possible to efficiently solve the MIP model and obtain the optimal solution. Figure 1 illustrates the solution found. The blue and dotted green arrows represent loaded-hoist movements and empty-hoist movements respectively. There are moves, where the hoist has more time to make the move in the schedule than it really needs. In this case we may decide whether to stop and wait over a tank or move slower. The figure illustrates the later choice. The black horizontal lines depict the intervals, where a carrier is in the given tank. Tanks with a non-continuous black line have carriers inside at the start of the cycle.
The solver was also evaluated on additional instances from [2]. For both the 16-tank "Zinc" instance and the 15-tank "ligne2" instance, the optimal cycle length for the simple-cycle case was successfully determined with negligible computational times.

5 Directions for Further Study

The methods and ideas used in the single-hoist CHSP provide a foundation for future studies. The multi-hoist CHSP is a natural extension of the problem, in which multiple hoists must be coordinated to complete all tasks while ensuring that no collisions occur. Further complications include introducing different part types with different soaking time

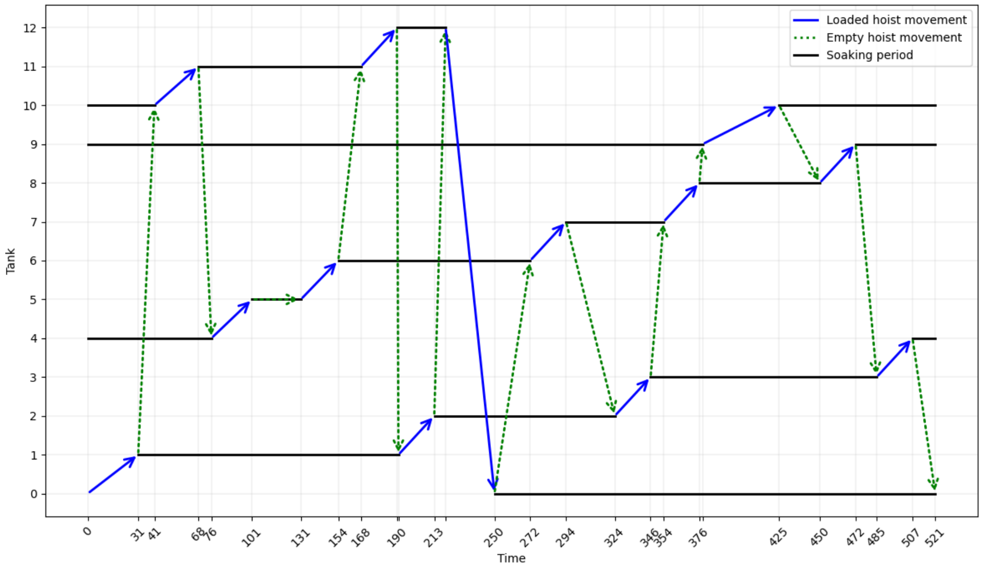

Figure 2: 12 tanks single-hoist CHSP example

limits, duplicated tanks to speed up processing in tanks with long minimum soaking time and extra constraints regarding the loading and unloading of carriers. A more distant goal could be the analysis of dynamic HSP [3], which consists of computing a new schedule of all the operations each time a part enters the line, while respecting all of the constraints.

References

[1] Phillips, L. W., and P. S. Unger. “Mathematical Programming Solution of a Hoist Scheduling Program.” A I I E Transactions 8 (2): 219–25. (1976).

[2] Emna Laajili. Modélisation et algorithmes pour le dimensionnement et l’ordonnancement cyclique d’atelier de traitement de surface. Automatique / Robotique. Université Bourgogne Franche-Comté, (2021).

[3] Manier, Marie-Ange, and Christelle Bloch. "A classification for hoist scheduling problems." International Journal of Flexible Manufacturing Systems 15.1: 37-55. (2003).