Epidemics on Hypergraphs

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Individual Project I

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Introduction

- Hypergraph structure
- CTMC description
- Master equations
- Bottom-up model
  - $\langle I_j \rangle(t) =$ probability that node j is infected at time t.

Models considered

Hypergraph on 3 nodes, hyperedge of order 3 being present.

If the hypergraph contains no edges, the resulting system is

$$\begin{aligned}
\langle \dot{I}_1 \rangle &= \beta(1 - \langle I_1 \rangle)\langle I_2 \rangle\langle I_3 \rangle - \gamma\langle I_1 \rangle,\\
\langle \dot{I}_2 \rangle &= \beta(1 - \langle I_2 \rangle)\langle I_1 \rangle\langle I_3 \rangle - \gamma\langle I_2 \rangle,\\
\langle \dot{I}_3 \rangle &= \beta(1 - \langle I_3 \rangle)\langle I_1 \rangle\langle I_2 \rangle - \gamma\langle I_3 \rangle.
\end{aligned}$$

Lumping of states leads to

$$\dot{x} = \beta(1 - x)x^2 - \gamma x.$$

For one, two and three edges respectively, we have

1. $$\dot{x} = \tau(1-x)x + \beta(1-x)xy - \gamma x,$$
$$\dot{y} = \beta(1-y)x^2 - \gamma y,$$

2. $$\dot{x} = \tau(1-x)y + \beta(1-x)xy - \gamma x,$$
$$\dot{y} = 2\tau(1-y)x + \beta(1-y)x^2 - \gamma y,$$

3. $$\dot{x} = 2\tau(1-x)x + \beta(1-x)x^2 - \gamma x.$$

Bifurcation diagrams

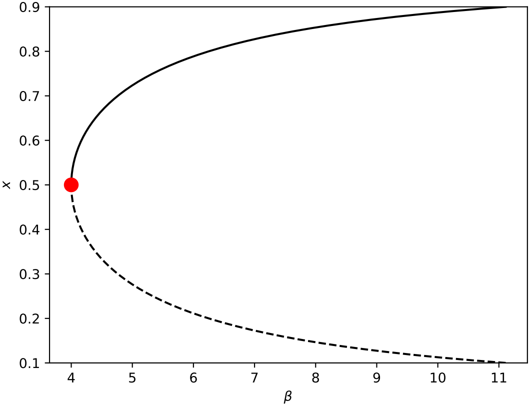

Figure: Bifurcation diagram of the zero edge equation in (β, x) space.

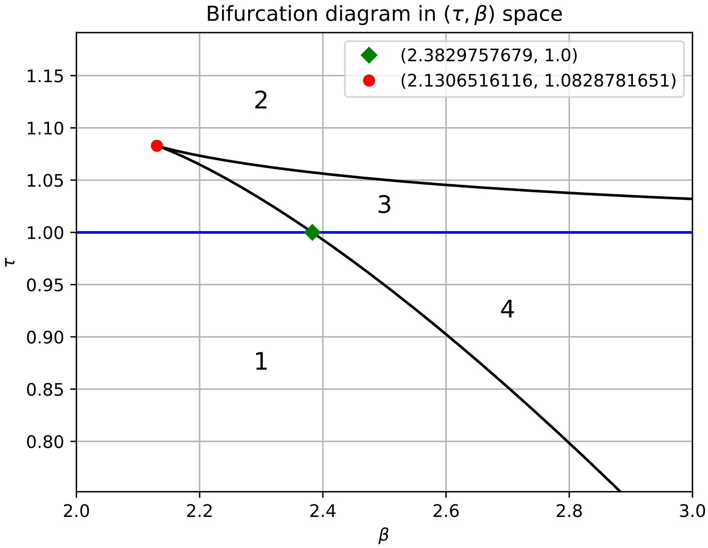

Figure: Bifurcation diagram of the one edge system in (β, τ) space.

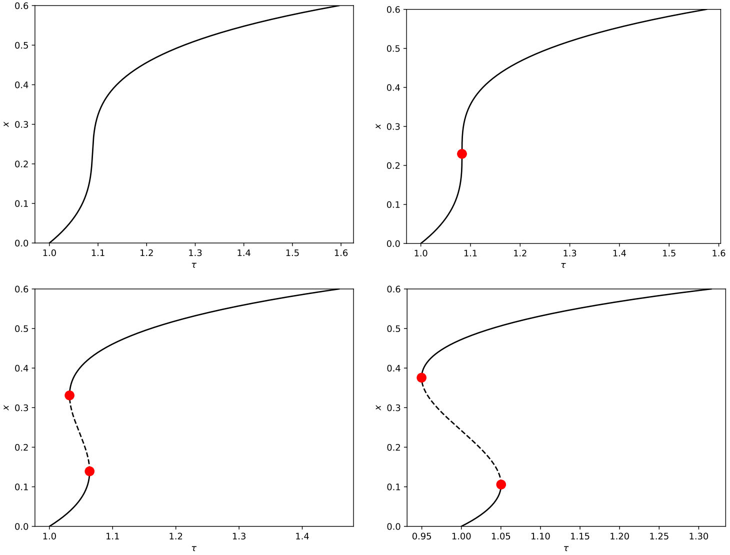

Figure: Bifurcation diagrams of the one edge system in (τ, x) space for various values of β.

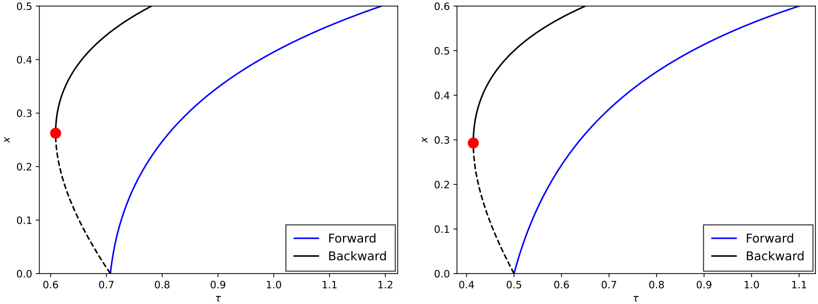

Figure: Bifurcation curves of the two and three edge systems in (τ, x) space.