Cooperative infinite games

Dániel Hegedűs
Supervisor: Benedek Kovács

Directed Studies 2

The original problem

Problem

There are 10 *prisoners standing in a line, all facing towards the same end of the line. Each prisoner is given a hat which is either red or blue. Each prisoner can see all of those prisoners' hats who are in front of them. They guess the colour of their own hat in the order they are standing, starting with the prisoner who sees everyone else. They hear each other's guesses and can use this information to guess their own colour. The question is to determine the maximum number of correct guesses that can be guaranteed, and to give a strategy achieving it. The prisoners can fix a strategy before they get their hats, but must not communicate after that (except hearing each other's guesses).*

Generalization

- The colour set can be any set C with cardinality κ
- Prisoners are standing on a subset S of $\mathbb{R}$
- Number of prisoners can be infinite, even uncountable
- The game may be continuous, for example for $S = (0, 1)$

Some easy results

Theorem

For a fixed n, let S be the non-negative integers not greater than n, furthermore let C be a set with an arbitrary cardinality κ. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

Theorem

Let S be the non-negative integers, furthermore let C be $\{0, 1\}$. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

The main method

- Let two colour-sequences be equivalent, if they only differ at finitely many places
- From each equivalence class, the prisoners pick a representative (before getting the hats)
- The starting prisoner (if exists) tells information about the places of differences compared to the representative

Some interesting results

Theorem

Let S be the non-negative integers, furthermore let C be $\{0, 1\}$. Also suppose that the prisoners are deaf, meaning that they cannot hear the guesses of the prisoners standing on smaller numbers. We state that it is possible to guarantee that at most only finitely many prisoners are incorrect, but for any non-negative k, it cannot be guaranteed that at most k prisoners are incorrect.

Theorem

Let $S = [0, 1]$, furthermore let C be a set with an arbitrary infinite cardinality κ. Also suppose that the prisoners are deaf, meaning that they cannot hear the guesses of the prisoners standing on smaller numbers. We state that it is possible to guarantee that at most countably infinitely many prisoners are incorrect. Furthermore at the same time, there exists a $t \in [0, 1)$ such that every prisoner in $[t, 1]$ guesses correctly.

Results from Directed Studies 1

Theorem

Let $S = \{0\} \bigcup \{\frac{1}{n} : n \in \mathbb{Z}, 0 < n\}$, furthermore let C be $\{0, 1\}$. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

Theorem

Let $S = \{\frac{1}{n} : n \in \mathbb{Z}, 0 < n\}$, furthermore let C be $\{0, 1\}$. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

New results

Theorem

Let S be well-ordered, furthermore let C be $\{0, 1\}$. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

Theorem

Let S be backwards well-ordered, furthermore let C be $\{0, 1\}$. We state that it is possible to guarantee that at most only 1 prisoner is incorrect and it cannot be guaranteed that 0 prisoners are incorrect.

Theorem

Let $S = \{0, \ldots, n\}$ and have k colours and $n \geq 2\ell + 1$ players with ℓ impostors among them. We state that it is possible to guarantee that at most $2\ell + 1$ prisoners give incorrect guesses.

Thank you for your attention!

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Taken from LaTeX package realhats.