Chromatic number of odd distance graphs on a circle

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MOTIVATION

$\boldsymbol{G^{odd}}$:

- V(G): points of the Euclidean plane
- E(G): odd length segments

What is the chromatic number of the odd distance graph $\boldsymbol{G^{odd}}$?

Davies: Can’t be colored with finite many colors→What about circles on the plane?

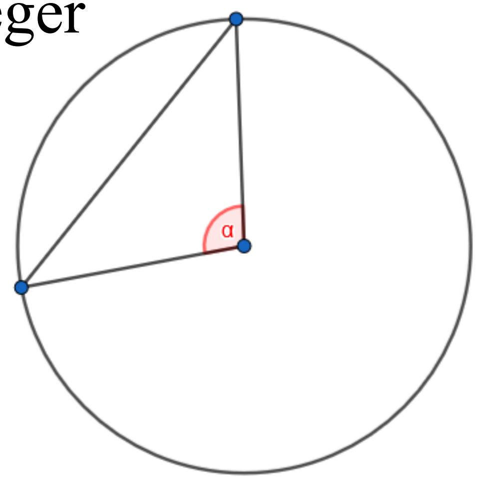
OVERVIEW

- G^r:
  - $V(G^r)$: points of a circle
  - $E(G^r)$: odd distances between vertices
- Main question: How many colors are needed to color G^r?
- Depends largely on the radius, only countable many cases are interesting
- $r = p\sqrt{Q}, \ \ p \in \mathbb{Q}, \ Q$ square free integer

Residual of an angle

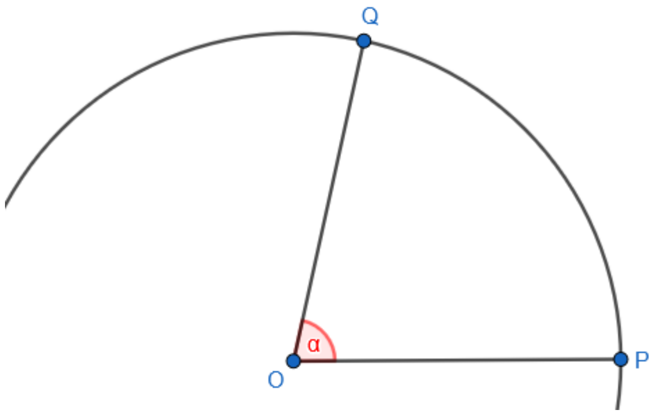
- Let α be an angle, such that:
  - $\sin(\alpha) = p\sqrt{D}$, where p is rational, and D is a square-free integer
  - $\cos(\alpha)$ is rational
  - Then α has a residual D
- Most of the angles doesn’t have a residual
- Central angle of a chord has a residual
- For a given residual these angles are closed under addition
- Cycle:

$$\sum_i \alpha_i = 2k\pi$$

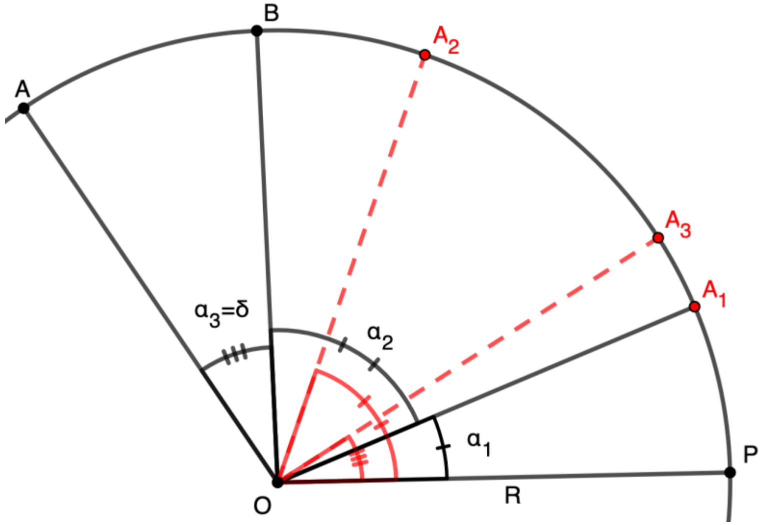


THEOREM 1

- Fix a P point on the circle, and for every point Q let the central angle be POQ∡.



It is sufficient to properly k-color the points whose central angle has a residual D for each D square-free integer.



Theorem 2

For $D \neq D_r$ the graph G_D is 2-colorable, where G_D is the subgraph of G^r induced by the points with residual D, and $r = p\sqrt{D_R}$.

Theorem 3

Given a circle of radius $\frac{a}{b}\sqrt{D_R}$, the subgraph of G^{odd} on the circle G_{D_R} is trivial, if

- $2 \nmid$ b and $D_R \not\equiv 3$ or $7 \mod 8$
- $2 \mid$ b but $4 \nmid$ b and $D_R \not\equiv 1$ or $5 \mod 8$

FURTHER RESEARCH

- In the next semester my aim is to examine the chromatic number of G_{D_R} for various R, and find useful properties, that can help in the investigation of this topic.
- Searching for non-trivial cases with computer

Thank you for listening!