The nucleolus and related notions in cooperative games

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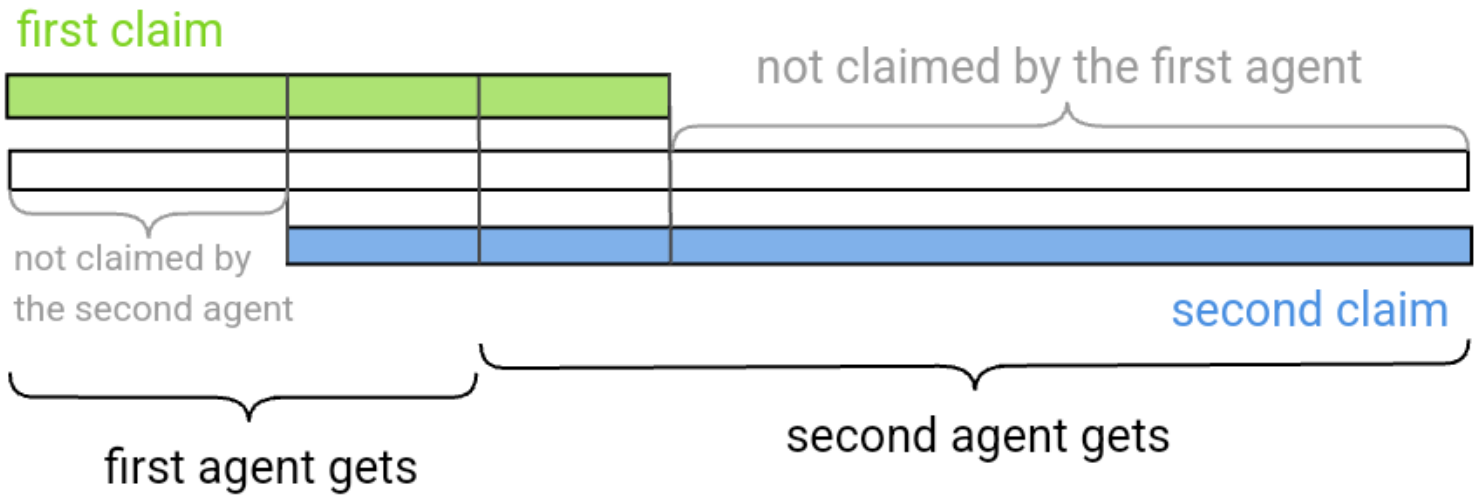
Cooperative games

- player set: $N = \{1, 2, \dots n\}$
- coalition: $S \subseteq N$
- characteristic function: $v : 2^N \to \mathbb{R}$
- allocation: $x \in \mathbb{R}^n$, $I^*(N, v)$, $I(N, v)$, $C(N, v)$
- profit: $p(S, x) := x(S) - v(S)$, profit vector: $\Theta(x) \in \mathbb{R}^{2^n}$ which contains all the profit values of different coalitions in non-decreasing order
- **nucleolus**: $N(v) = \{x \in I(N, v) | \quad \Theta(y) \preccurlyeq \Theta(x) \quad \forall y \in I(N, v)\}$
- examples: horse fair, glove market

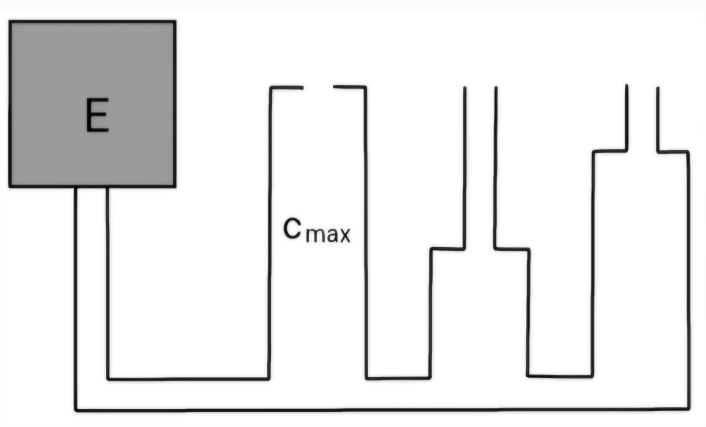
Bankruptcy problem

- firm's liquidation value: $E \in \mathbb{R}_+$
- claims: $c \in \mathbb{R}^n_+$
- $\mathbb{B}$ denotes the set of such (c, E) problems
- $x \in \mathbb{R}^n_+$ is a solution if $\sum_{i=1}^n x_i = E$
- rule: $r : \mathbb{B} \to \mathbb{R}^n$

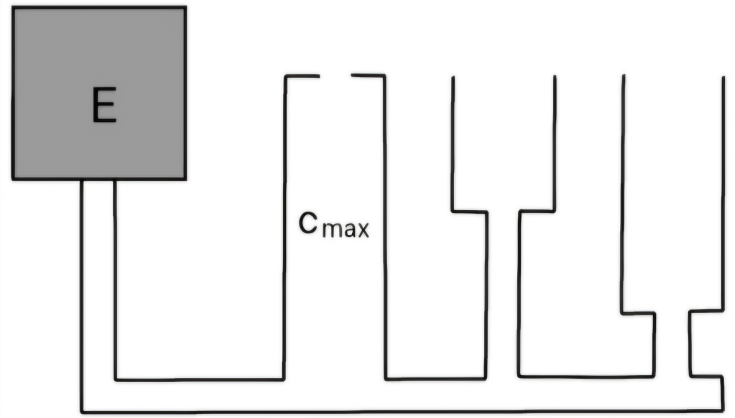


1. ábra. Talmud rule

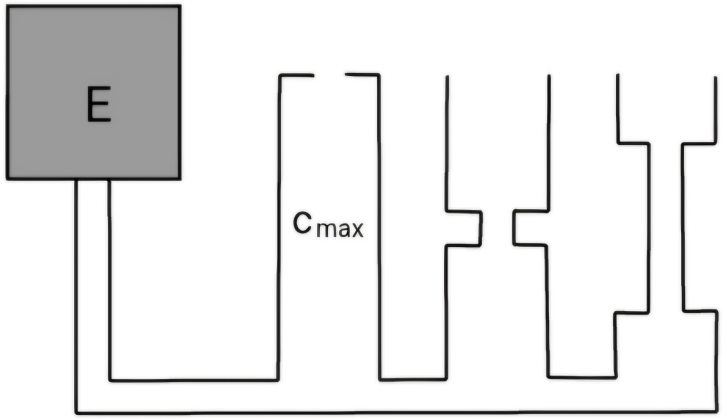
Hydraulic rationing



equal awards



equal losses



Talmud rule

Overview

Kaminski: In the two-person bankruptcy problem, the corresponding talmudic hydraulic system's x solution is CG-consistent.The following theorem shows why it is worth dealing with the Talmud rule.
Aumann, Mascher: The CG-consistent solution to the bankruptcy problem is the nucleolus of the corresponding cooperative game.

future research: other special cases where the nucleolus can be calculated in similarly simple ways