GRAPHS WITH INTEGER EDGES ON THE PLANE

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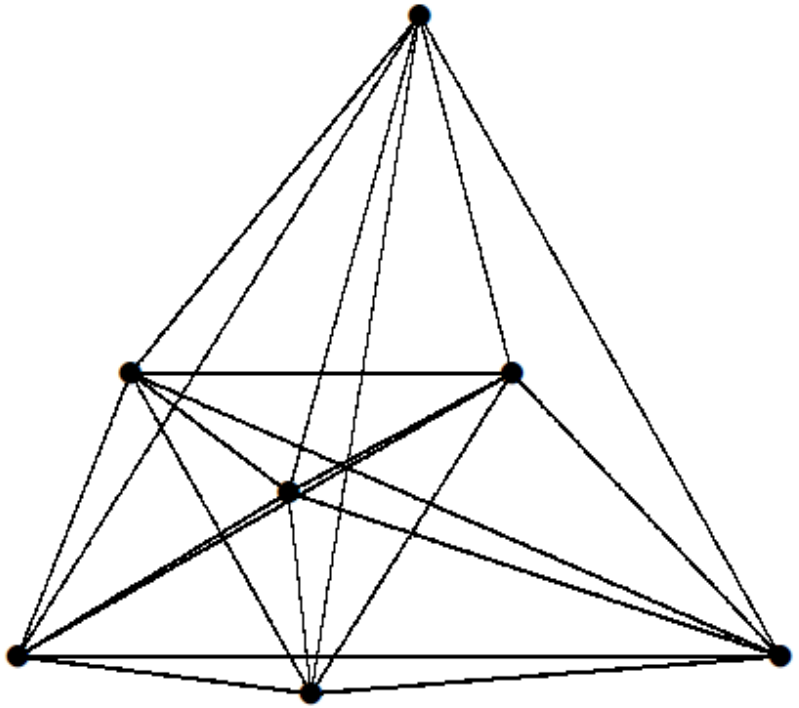
Motivation

- Connected topics:
  - Erdős-Ulam conjecture: There exists a dense set in the plane in which any two distances are rational.
  - Harborth conjecture: Every planar graph has a planar drawing in which every edge is a straight segment of integer length.
- Find new integer point sets with no 3 points on a line and no 4 points on a circle

Overview

- Complete graphs = integer point sets
- Biggest one: 7 points
- Is there one consisting 8?
- Needs a lot of computational power

n	Smallest diameter
3	1
4	8
5	73
6	174
7	22270

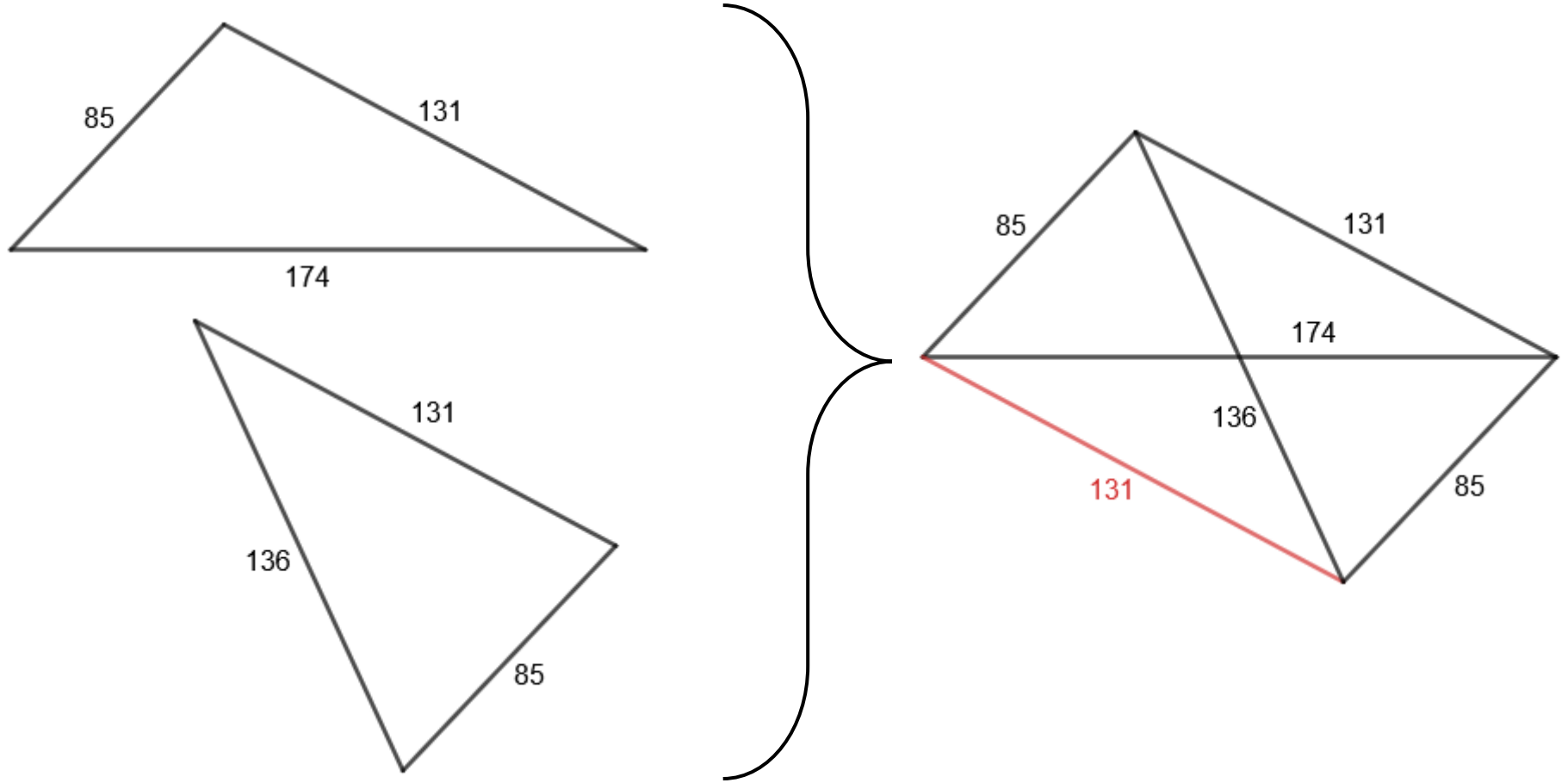


$\left(\frac{0}{1}, \frac{0}{1}\sqrt{2002}\right)$
$\left(\frac{22270}{1}, \frac{0}{1}\sqrt{2002}\right)$
$\left(\frac{26127018}{2227}, \frac{932064}{2227}\sqrt{2002}\right)$
$\left(\frac{245363}{17}, \frac{3144}{17}\sqrt{2002}\right)$
$\left(\frac{17615968}{2227}, \frac{238464}{2227}\sqrt{2002}\right)$
$\left(\frac{56068}{17}, \frac{3144}{17}\sqrt{2002}\right)$
$\left(\frac{19079044}{2227}, -\frac{54168}{2227}\sqrt{2002}\right)$

First found integral point set with 7 points

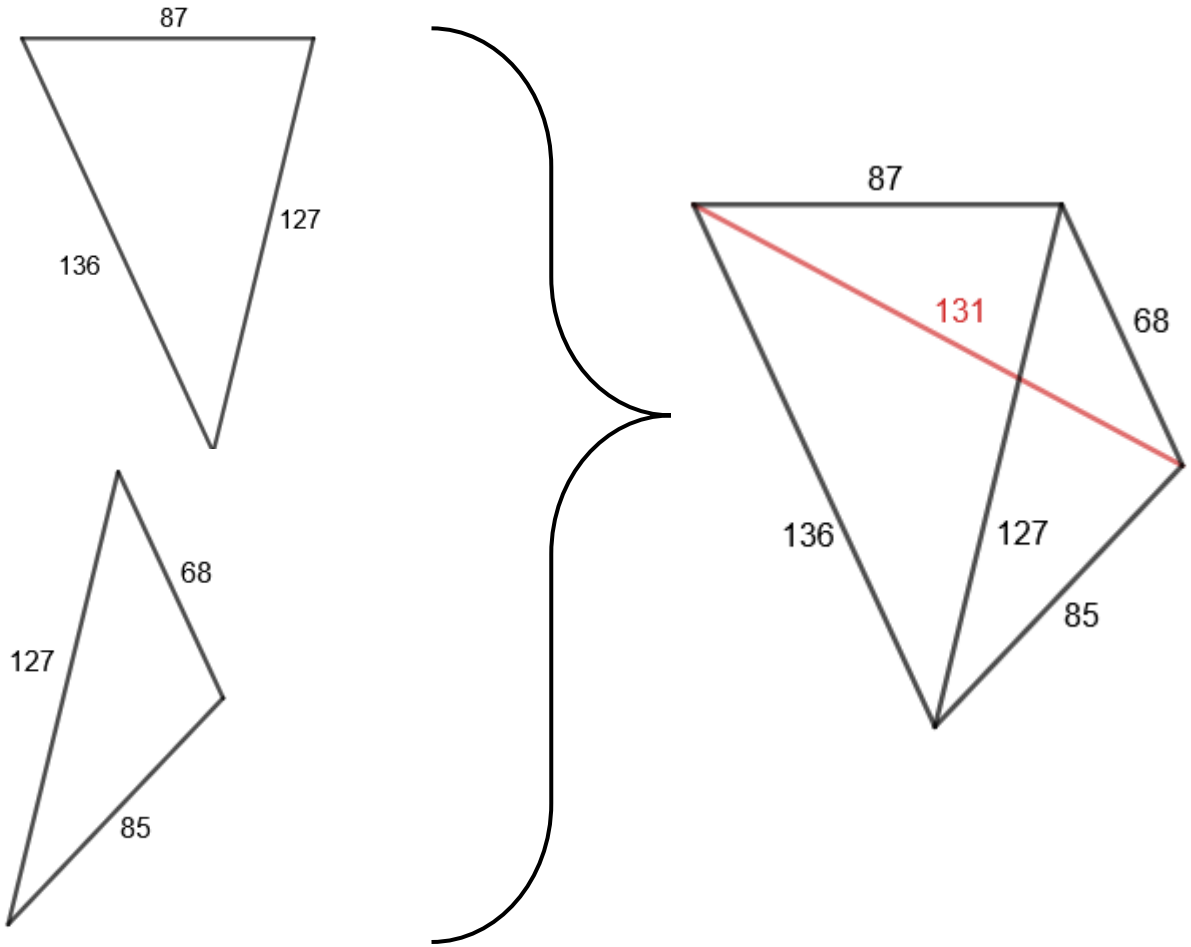
EXHAUSTIVE SEARCH

I.



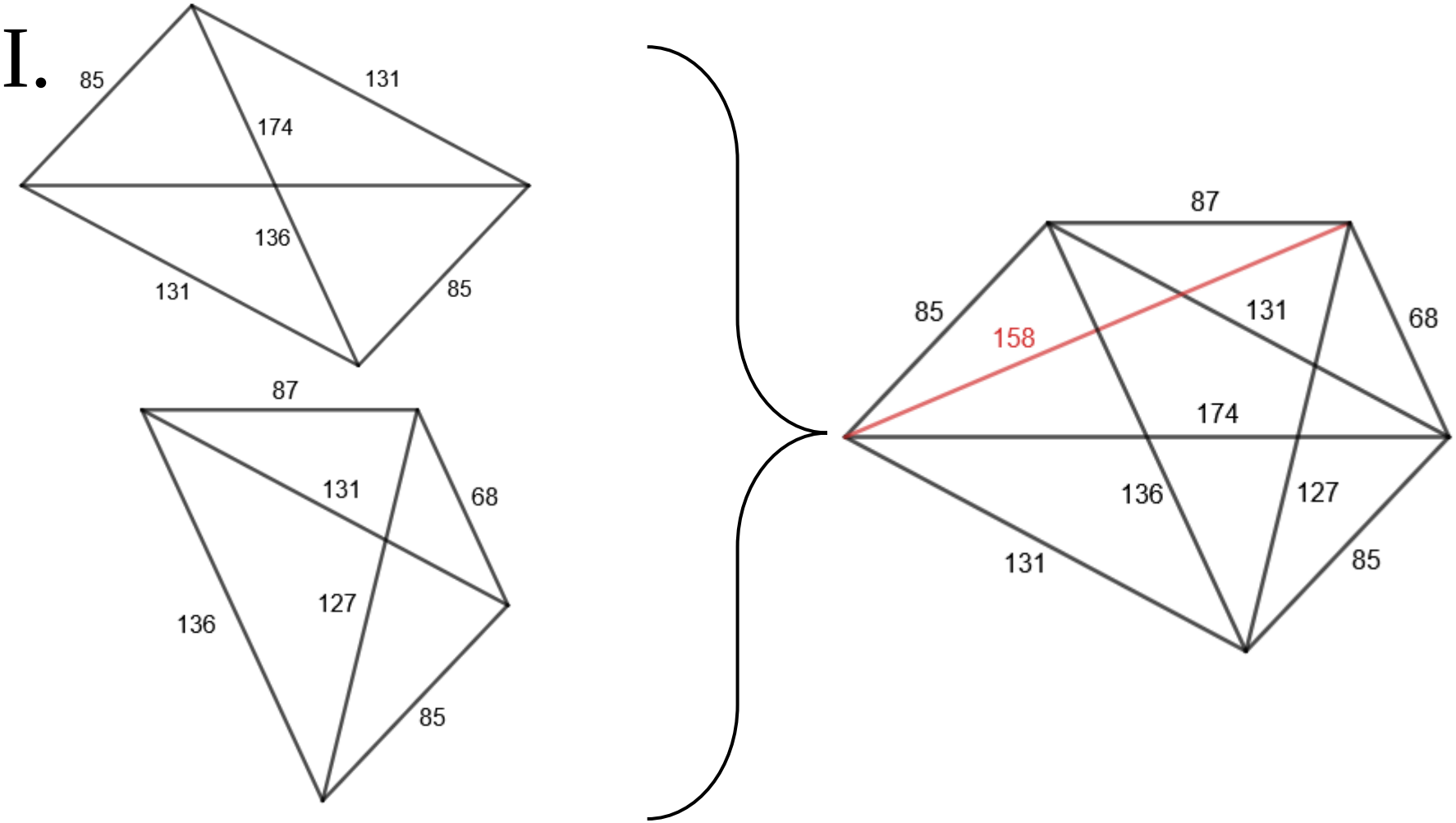
EXHAUSTIVE SEARCH

II.



EXHAUSTIVE SEARCH

I+II.



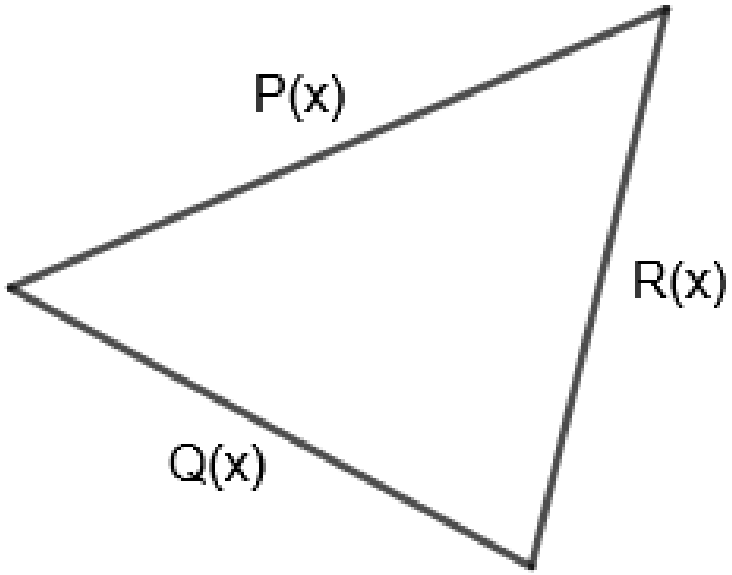
CHARACTERISTIC

- Area of a triangle: $A = l\sqrt{k}\ \ l \in \mathbb{Q}, k \in \mathbb{Z}, k$ square free
- Characteristic: k
- Statement: In an integer point set every triangle has the same characteristic. → Faster algorithm

POLINOMIAL CHARACTERIZATION

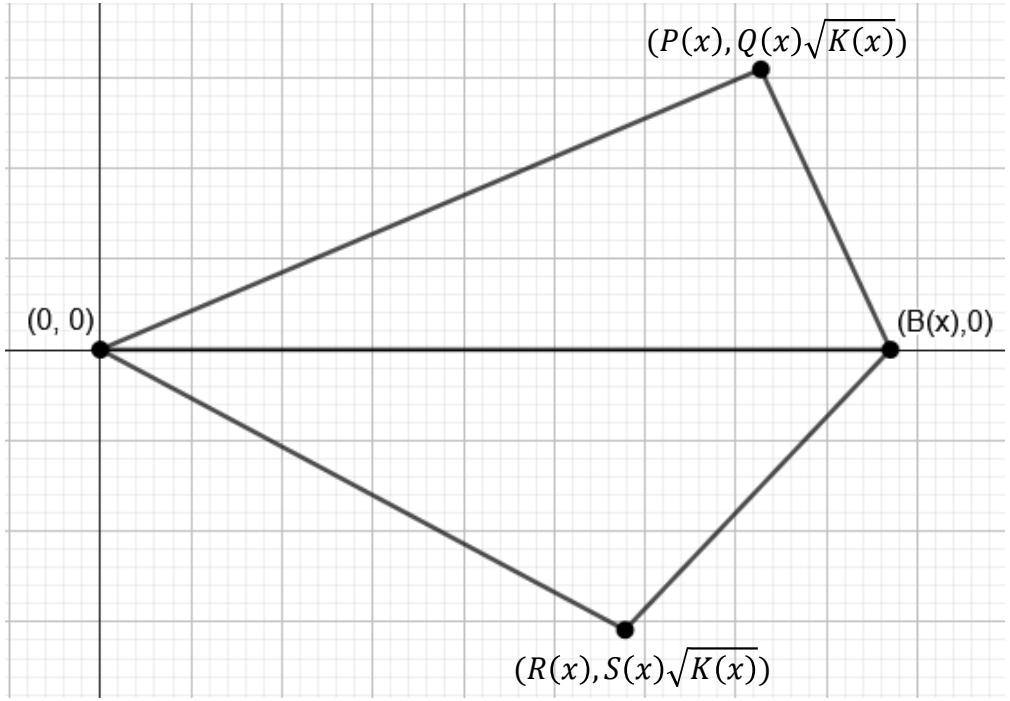
- Edges: $P(x) \in \mathbb{Z}[x]$
- Characteristic: $K(x) \in \mathbb{Z}[x]$,

monic, square free



COORINATE-SYSTEM

- We can place the points in a coordinate-system where all point has coordinates in the form $\left(p(x), q(x)\sqrt{k(x)}\right)$, where k(x) is the characteristic.
- If k(x)=1, enough to search integer coorinates



FURTHER RESEARCH

- Examine that if we fix a characteristic, then how can we generate parametric integer point sets, and hopefully I find new parametric constructions that may generate new integer point sets
- Creating new parametrized sets using the method of uniting smaller ones

THANK YOU FOR LISTENING