Generating Small Graphs up to Isomorphism

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Goal of the project

Specialized graph iterator

- No isomorphic graphs
- Given number of nodes
- Other, preferably vertex-hereditary graph properties

Property examples

Vertex-hereditary:

- K_3-free
- C_4-free
- bipartite
- upper bounds for degrees / number of edges
- (and/or) combinations of the above

Not inherently vertex-hereditary

- connected
- lower bounds for degrees / number of edges
- negations of the above

The basics of the graph-generating algorithm

A search-tree of graphs:

- the root is $G = (\emptyset, \emptyset)$
- the leaves are the output graphs

Let H be one of $G's$ children.

- $V(H) = V(G) + w$
- $E(H) \supseteq E(G)$ and all new edges are incident to w

Genertaing graphs of order 3

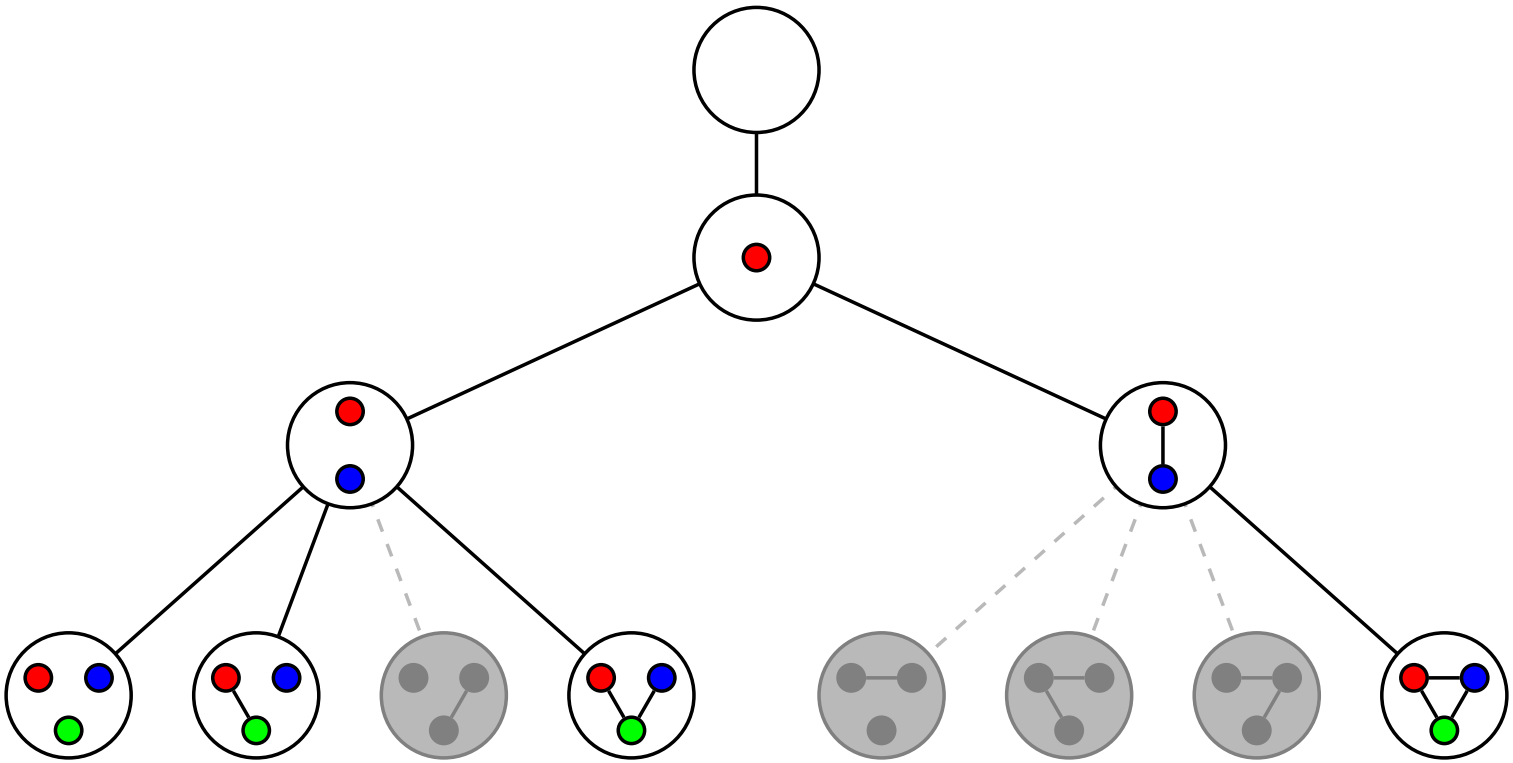


Figure: The search tree of the unlabeled generator

How to choose the children of G

$H_1 \nsim H_2$ for graphs on each level

- Handled by canonization
  - No two children with new neighborhoods in one *set-orbit*
  - New vertex is in one orbit with the *canonical labeling*'s maximal vertex

Keep graph properties

- Handled by graph filters
  - Check if properties still hold
  - Check if properties can still be satisfied

The canonization algorithm

A search-tree of *ordered partitions* of $V(G)$

- The root is the trivial partition
- The leaves are discrete partitions $\rightarrow$ permutations
- A child-partition is always a *refinement* of its parent.

We choose the leaf that leads to a "minimal graph" when permuting G with it, and make note of automorphisms found along the way.

Need: Isomorphic graphs choose from same pool of graphs

How to choose children partitions

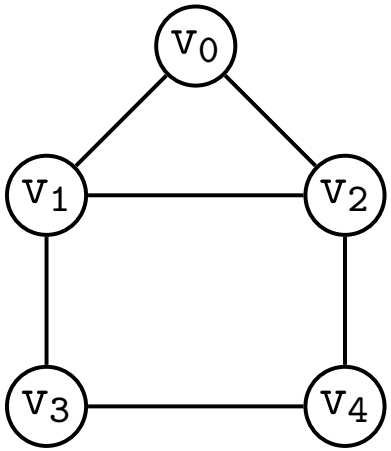
Brute-force:

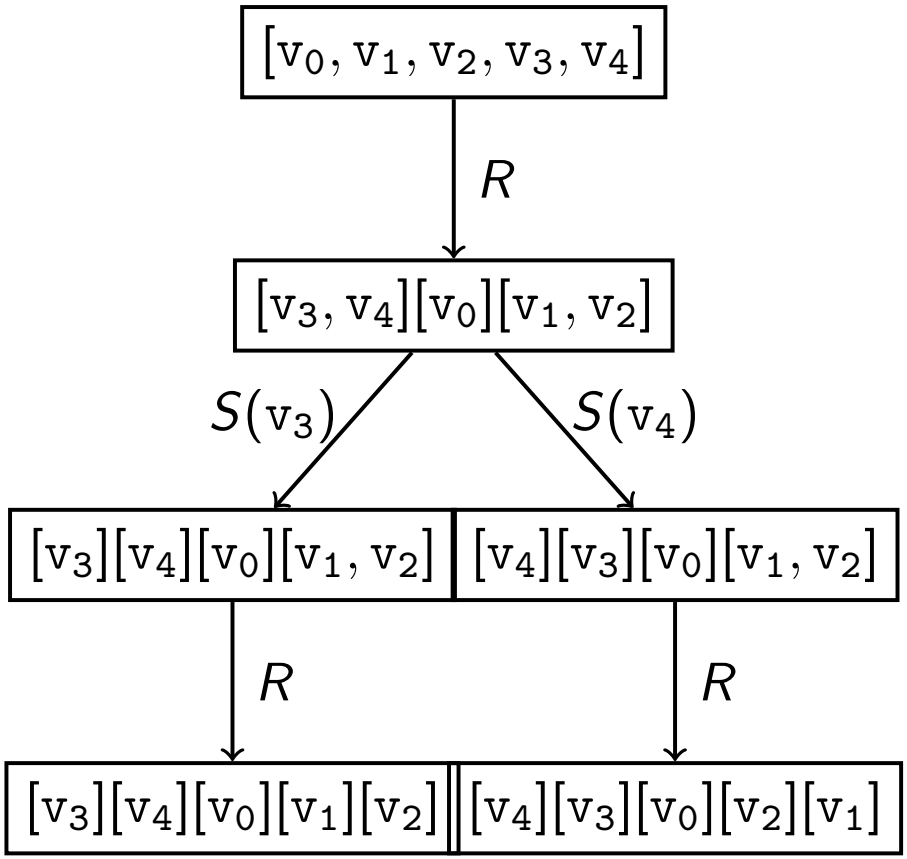
- pick first non-trivial cell C, place the element e in front for all $e \in C$
- leads to all $n!$ permutations

Smarter:

- use degree-properties from the graph to refine partitions before generating children like above
- leads to much, much smaller search trees

Partition search-tree example





Our work this semester

- Working canonizer, set-orbit calculator
- Working generator using the above
- Working filter base, example filters