The price-collecting traveling salesman and related problems

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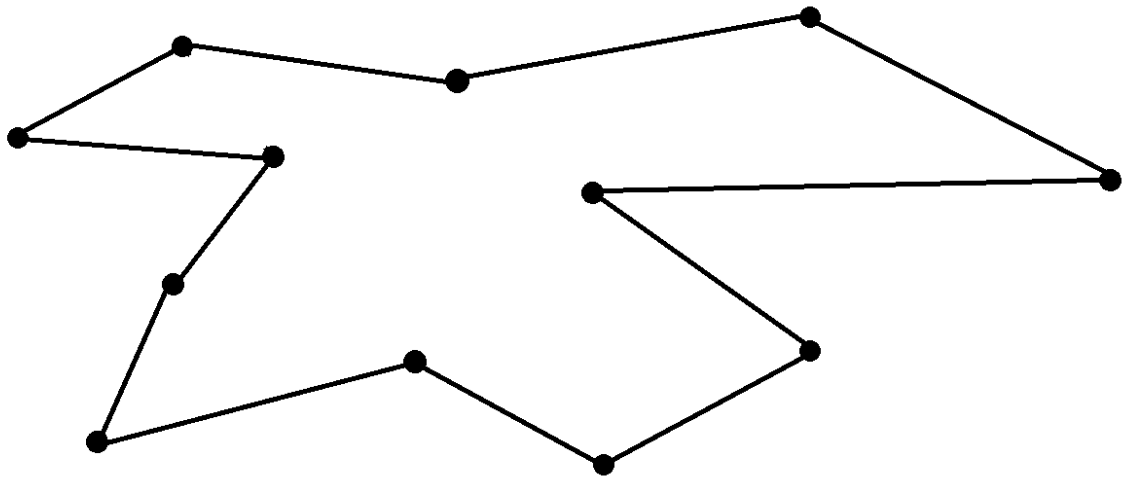
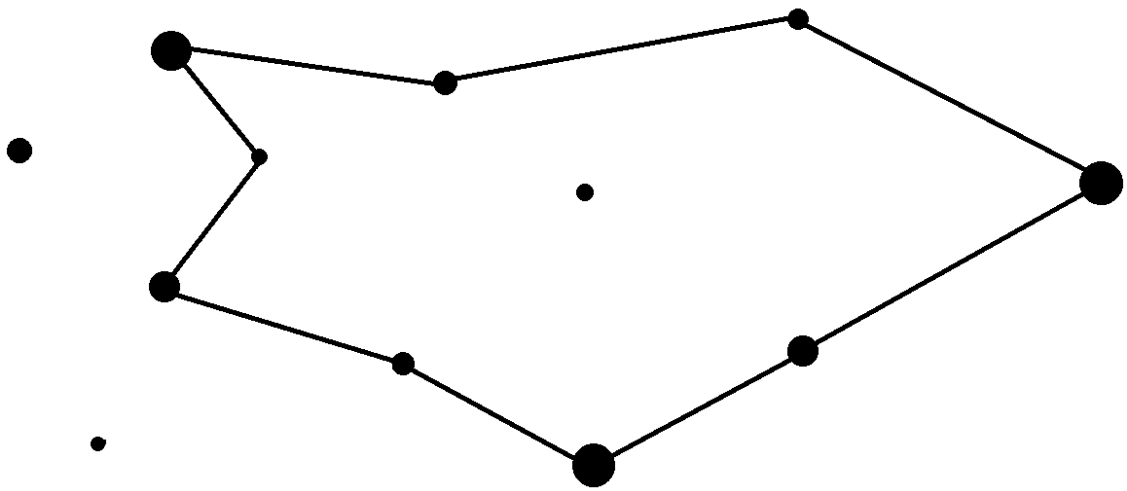
30 May, 2024

Example of a tour in TSP



Example of a tour in PCTSP



Definition

Given a complete graph $G = (V,\ E)$, a root $r \in V$,
$c_e \geq 0\ \forall e \in E$ metric lengths of the edges and
$\pi_v \geq 0\ \forall v \in V \backslash \{r\}$ vertex weights,
the price-collecting traveling salesman problem is to find a
cycle $C = (V_C,\ E_C)$ in G, so that $r \in V_C$, and
$\sum_{e \in E_C} + \sum_{v \in V \backslash V_C}$ is minimal.

LP relaxation

$$min \sum_{e \in E} C_e x_e + \sum_{v \in V} \pi_v (1 - y_v)$$

$$\begin{aligned}
x(\delta(v)) &= 2y_v \quad \forall v \in V \backslash \{r\} \\
x(\delta(r)) &\leq 2 \\
x(\delta(S)) &\geq 2y_v \quad \forall S \subseteq V \backslash \{r\},\ v \in S \\
y_r &= 1 \\
x_e &\geq 0 \quad \forall e \in E \\
y_v &\geq 0 \quad \forall v \in V
\end{aligned}$$

A heuristic algorithm

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- Choose the best tour

Improvements

1. **Deleting vertices**
   Given a tour, check for each vertex if the cost can be decreased by deleting that vertex.

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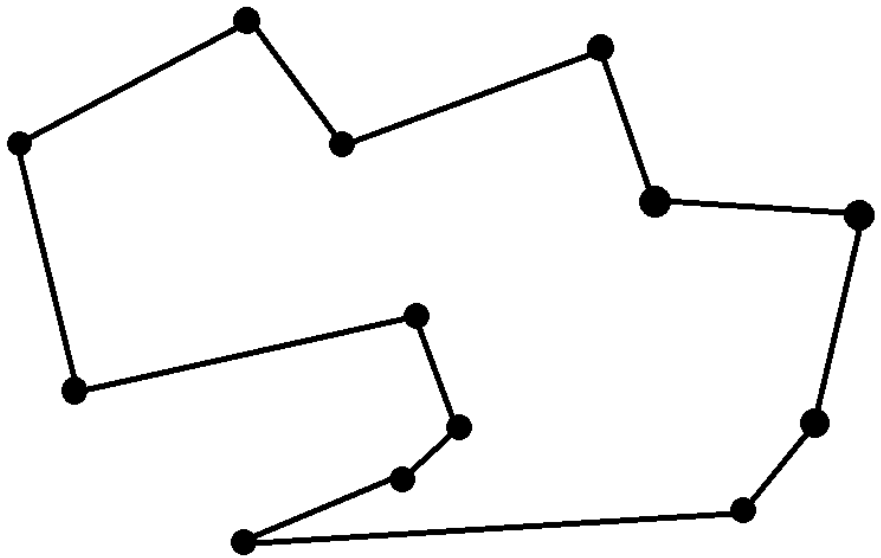

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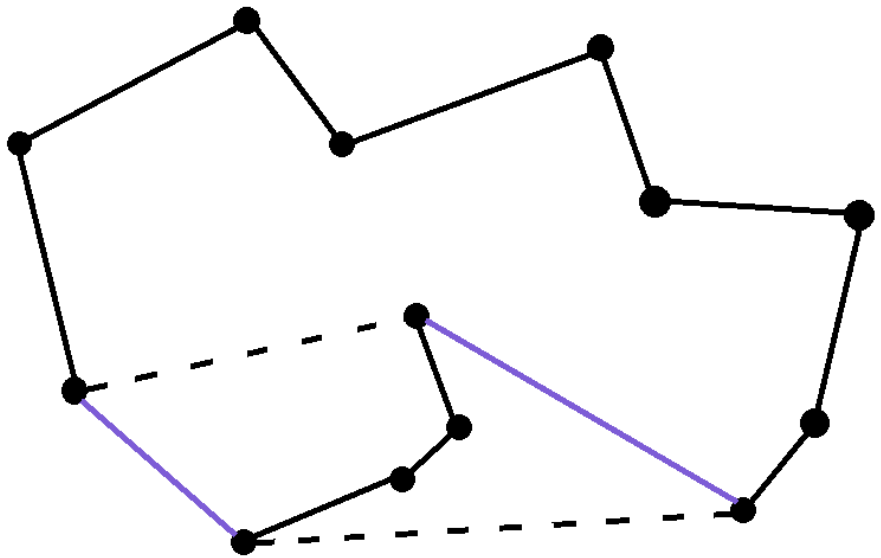

Testing on random graphs

Generating graphs:

- fixed $n = 100$ vertices
- vertex weights: random integers between 0 and 100
- edge lengths: random coordinates between 0 and 800, euclidean distances, rounding

Test results

Area size	Heuristic	Length 1	Deletion first	Length 2	Switch first	Length 3
500.0	1474.18	97.85	961.93	79.57	1203.52	83.10
550.0	2096.05	96.02	1605.30	76.99	1803.82	81.24
600.0	2633.75	93.09	2159.29	72.81	2297.20	77.70
650.0	3014.64	90.25	2556.36	70.03	2627.45	73.99
700.0	3515.59	85.21	3102.10	64.51	3104.95	68.56
750.0	3953.21	77.60	3549.42	58.83	3545.26	62.00
800.0	4352.13	60.53	4070.30	45.40	3996.66	47.76
850.0	4552.50	40.87	4357.64	30.35	4303.23	31.98
900.0	4724.41	25.40	4621.91	19.15	4576.82	20.14
950.0	4788.13	14.26	4747.75	10.50	4703.43	11.04
1000.0	4808.88	8.82	4787.54	6.64	4758.12	7.01

Future plans

- Calculate lower bound for PCTSP from the LP relaxation
- Implement and test more advanced algorithms

Thank you for the attention!