Cyclic orderings and the generalized Gabow conjecture

Jánosik Áron
Advisor: Bérczi Kristóf

While studying the structure of symmetric exchanges in matroids, Gabow [5] formulated a beautiful conjecture on cyclic orderings of matroids. The question was later raised again by Wiedemann [10] and by Cordovil and Moreira [4].

Conjecture 1 (Gabow)**.** *Let B and B' be disjoint bases of the same matroid with rank r. Then there exists a sequence of r symmetric exchanges that transforms the basis pair (B, B') into (B', B).*

It is not difficult to see that the statement holds for strongly base orderable matroids. The conjecture was settled for graphic matroids [4,6,10], sparse paving matroids [1], matroids of rank at most 4 [8] and 5 [7]. A relaxation where the elements can be permuted in the union of the two bases was proposed in [9].

Conjecture 2 (Van den Heuvel and Thomassé)**.** *Let $B = \{s_1, \ldots, s_r\}$ and $B' = \{s'_1, \ldots, s'_r\}$ be bases of the same matroid. Then there is a permutation of the sequence $\{s_1, \ldots, s_r, s'_1, \ldots, s'_r\}$ in which any r cyclically consecutive elements form a basis.*

A natural generalization of Conjecture 2 is to start with k bases and to find a suitable cyclic ordering of the elements of these k bases combined. Surprisingly, no results are known for $k \geq 3$, not even for graphic matroids.

Problem 1. *Is it true that if the ground set of a matroid of rank r is the disjoint union of k bases, then there exists a permutation of the elements in which any r cyclically consecutive elements form a basis?*