Modelling and Optimisation of Optical Systems

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Introduction

Before this semester:

- program for modelling in Gaussian optics
- more complex code for more accurate simulation

This semester:

- perfected and expanded the C++ project
- added new features
- parameter optimisation:
  - achromatic lens
  - Tessar objective

Goals

The code is made for:

- modelling optical systems with flat and spherical surfaces,
- simulating light rays passing through them,
- analysing optical aberrations,
- visualisation (with the Cairo library),
- parameter optimisation.

Important addition

The refractive index of a material depends on the wavelength, therefore, it is important to consider the wavelength of the rays in the modelling.
This can be done using Sellmeier's dispersion formula:

$$n^2 - 1 = C_1 + \sum_{i=1}^{8} \frac{C_{2i}\lambda^2}{\lambda^2 - C_{2i+1}}$$

I parsed an online database, using Python, to get the C_i values.

Important addition

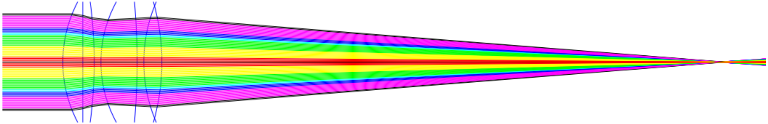


Figure: Light rays of different wavelengths passing through a Tessar objective

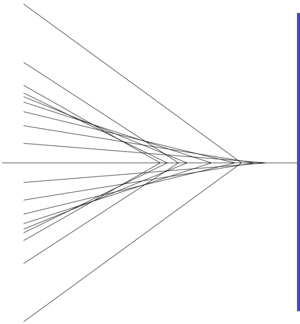
Optical aberrations

Optical aberrations occur because the paraxial theory is not an accurate model.

Types of optical aberrations:

- monochromatic aberrations,
- chromatic aberrations.

Monochromatic aberrations are caused by the geometric properties of the surfaces.



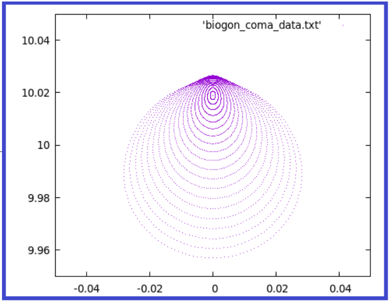


Figure: Spherical aberration and coma

Chromatic aberrations

Chromatic aberrations, on the other hand, occur due to dispersion.

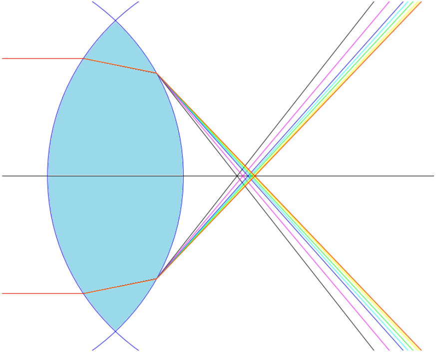


Figure: Chromatic aberration in a single equiconvex lens

Achromatic lens

Goal: reducing the difference between the focal points of two selected wavelengths.

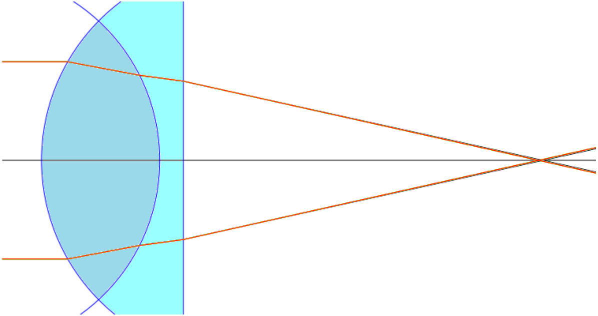


Figure: Chromatic aberration in an achromatic lens

Achromatic lens

Objective: minimise the difference between the focal point of the wavelengths 486.1 nm and 656.3 nm.

Constraints:

- Two elements:
  - a biconvex lens made of *K7*,
  - a plano-concave lens made of *F2*.
- The first lens should be 5 mm thick, the second 1 mm.
- The focal length of the achromat should be 100 mm.

Achromatic lens

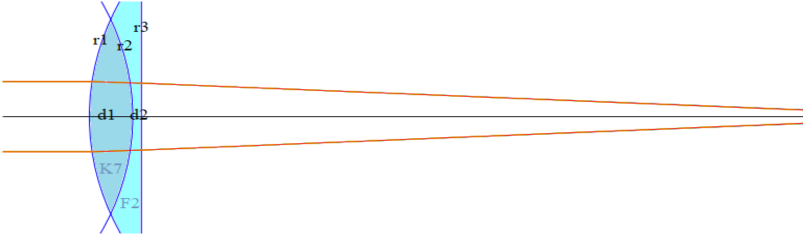


Figure: Achromatic doublet

- $d1 = 5\,\mathrm{mm}$
- $d2 = 1\,\mathrm{mm}$
- $r3 = \infty$
- $r1 = ?$
- $r2 = ?$

Goal

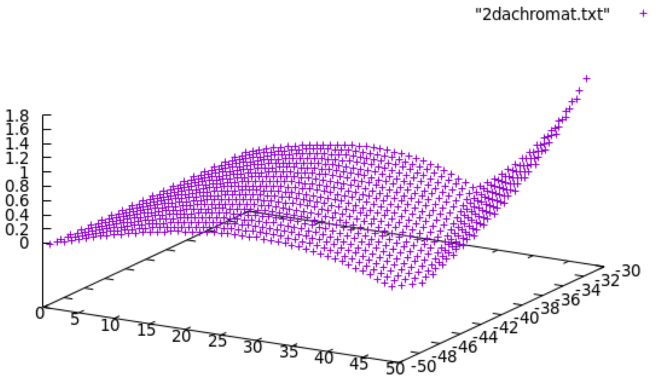


Figure: The difference between the focuses as a function of $r1$ and $r2$

Finding $r1$

We can find $r1$ for any given $r2$ using binary search.

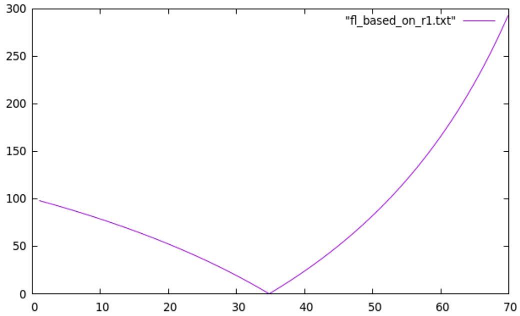


Figure: The amount by which the focal length deviates from 100 mm as a function of $r1$, where $r2$ is fixed to -22 mm.

Finding $r2$

To find $r2$, we need to minimise a quasiconvex function once again.

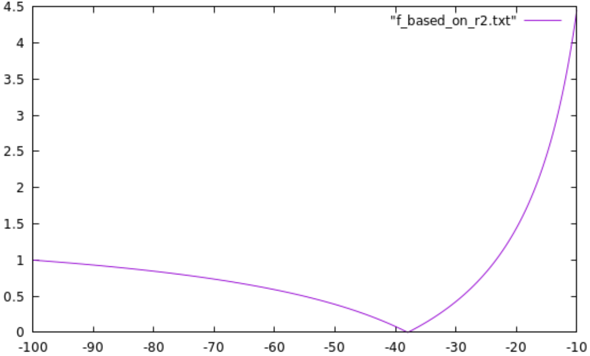


Figure: The target function as a function of $r2$

The Golden Section Method

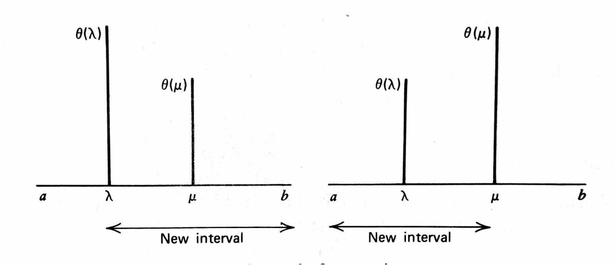


Figure: Reducing the uncertainty interval

The *Golden Section Method* is efficient for finding the minimum of a quasiconvex univariate function.
It does not require the use of derivatives.

The Golden Section Method

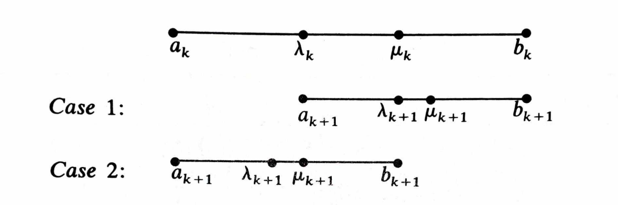


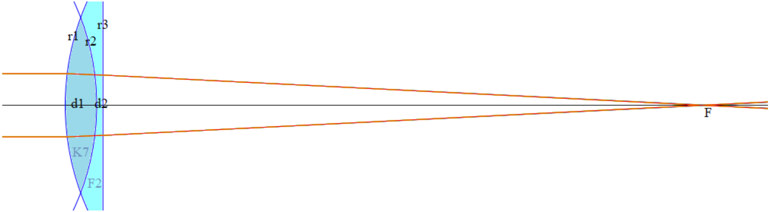
Figure: Choosing the next points for evaluation in the *Golden Section Method*

Results

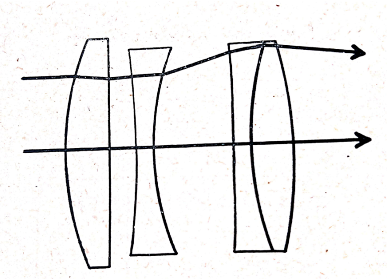
The resulting values for the two radii:

- $r1 = 40.0526\,\text{mm}$,
- $r2 = -37.968\,\text{mm}$.

The difference between the two focal points: $1.55 \cdot 10^{-7}$ mm. (Before the optimisation: 0.667 mm.)



Step 1: find similar materials



A lencsék balról jobbra, a belépő fénysugárral együtthaladva következnek.

	anyag	Az üvegfajta: közepes törésmutatója	Az üvegfajta: Abbe-száma (diszperzió)	A lencse: görbületi sugara mm.	A lencse: vastagsága ill. távolsága mm.
I.	Síkdomború lencse nehéz koronaüvegből	1·6113	58·4	21·5 ∞	3·3
	levegő				1·9
II.	kétszer homorú lencse báriumos flintüvegből	1·6046	43·9	74·2 20·8	1·1
	levegő				3·0
	a rekesz helye				
	levegő				3·0
III.	kétszer homorú lencse ú. n. rövid flintből	1·5211	51·8	111·3 25·2	1·1
	ragasztás kanada-balzsammal				
IV.	kétszer domború lencse nehéz koronaüvegből	1·6113	56·2	25·2 36·7	3·0

Step 2: Optimise the parameters

The parameters:

- radii: 7,
- distances between them: 6

$\rightarrow$ we need to optimise 13 parameters.

The objective is to minimise the

- axial chromatic aberration,
- spherical aberration,
- and the field curvature.

The focal length should be 100 mm.

Spherical aberration

y-axis: distance of the starting ray from the optical axis (mm), x-axis: how far from the focal point it intersected the optical axis (mm).

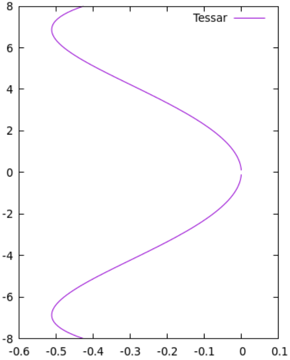


Figure: Spherical aberration in the Tessar objective

Field curvature

The amount of field curvature can be calculated analytically with the *Petzval-sum*:

$$\sum_i \frac{n_{i+1} - n_i}{r_i n_{i+1} n_i}$$

The task

The objective function that we need to minimise:

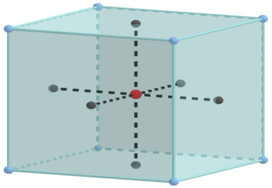
$$\textbf{obj} = \textit{constant} \cdot |\text{focal point of } 486.1\,\text{nm - focal point of } 656.3\,\text{nm}| + \textit{constant} \cdot \text{spherical aberration} + \textit{constant} \cdot \text{Petzval-sum}$$

The focal length has to be 100 mm ⇒ given the other 12 parameters, the last radius ($r7$) can be calculated using binary search.

⇒ This is a nonlinear optimisation problem in 12 dimensions, with a black-box function as the target.

Results

The original values turned out to be optimal ⇒ I changed the materials.
My method: multidimensional search along the coordinate axes inside a 12-dimensional brick.



The process is not complete, I managed to improve the objective function, but not by enough.

Future plans

I will continue the optimisation process in my thesis by implementing a more complex algorithm. My plan is to finish optimising the parameters of the Tessar objective and then make my program suitable for optimising other optical systems as well.

Thank you!