Parameters of percolation models

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Model

Definition

- Take $\mathbb{T}_N^d$ (as a CW-complex)
- Write an iid. $U([0,1])$ random number to each i-dimensional cell
- Fix a $p \in [0,1]$ and take the i-cells which has a number less than p.
- Now we have a filtration P_p of $sk_i T_N^d$ parametrized by $p \in [0,1]$.
- Let $\phi_{p_*} : H_i(P_p; \mathbb{Z}_2) \to H_i(\mathbb{T}_N^d; \mathbb{Z}_2)$ the induced homomorphism.

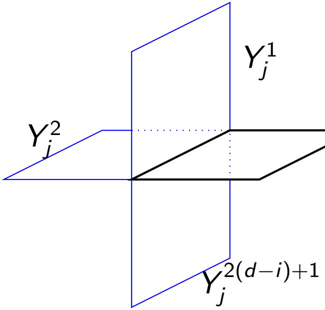
Questions

- For which p will be ϕ_{p_*} non-trivial/surjective/etc.?
- Can we effectively calculate ϕ_*?
- What happens with H_1?
- At the critical regime how the appearance of the different generator correlates?

Simplification

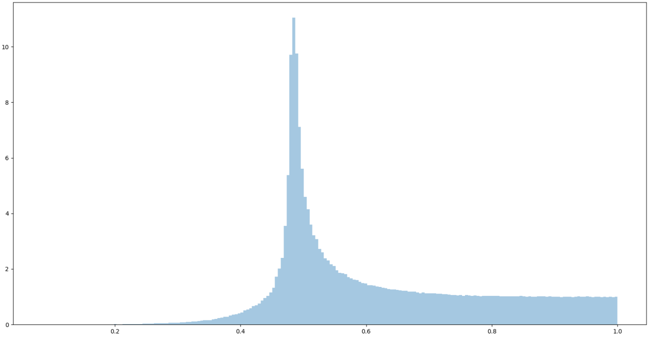
Idea: remove (delay the appearance of) the *i*-cells which cannot be in a cycle ie. has an unclosed boundary-component.

$$\max\{p(X), \max_{j=1}^{2i} \min_{k=1}^{2(d-i)+1} p(Y_j^k)\}$$



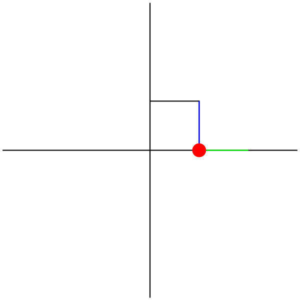
Simplification

Distribution of appearance times after the modification:



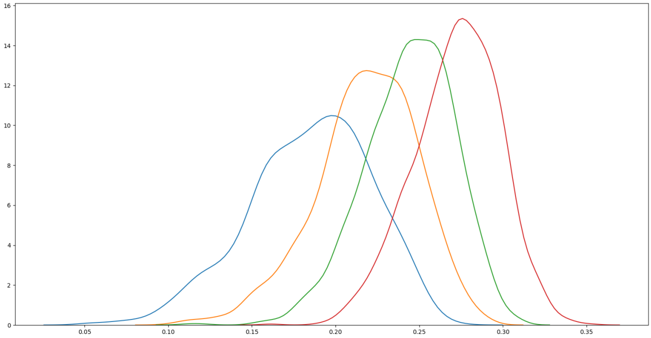
Simplification

Without case analisys we cannot go further.



Induced map on H_1

Time of the appearance of the generators ($N = 4$, $i = 2$).



Tubes and membranes

Induced map on H_1

Correlation of the appearance of the generators ($N = 4$, $i = 2$).

$$\begin{pmatrix} 1 & .0432 & .0448 & .0427 \\ .0432 & 1 & .0226 & .0501 \\ .0448 & .0226 & 1 & .0472 \\ .0427 & .0501 & .0472 & 1 \end{pmatrix}$$

Further questions

- With the modification of the filtration the independence of the appearance times no longer holds. But how strong the correlation deacay remains?
- There is a 'peak' in the density function of the new distribution at the critical probability. Is this true for other initial parameters as well?
- At low values of p, there are balls and tubes. Then membranes starts to appear. How these tubes relates to each other? And the membranes?