Cost-optimal, condition-based maintenance

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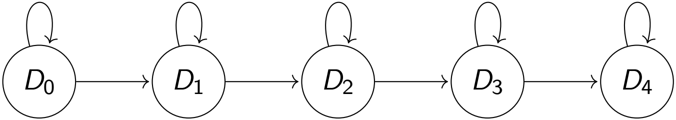
Introduction

- Suppose we have a degradation process that can be described by a discrete number of states. E.g. $D_0, \ldots, D_4$.
- Assume that the damage can stay hidden without inspection and it is unknown if an item is degrading or not.
- Suppose all items start from the damage-free state and accumulate damage over time.



- We are interested in establishing a cost-optimal inspection program to prevent catastrophic failures.

Model Setup

- Consider a continuous-time Markov chain $(X_t)_{t \geq 0}$ in a discrete state space $I = \{0, \dots, 4\}$ corresponding to states D_i and the progress is as it was indicated on the figure.
- Let Y_i be the waiting time in state D_i and suppose that $Y_i \sim exp(\lambda_i)$ for $i = 0, 1, 2, 3$.
- Assume that inspections have a constant false positive rate α and a perfect sensitivity.
- Denote by τ_i the age at the i^{th} inspection.
- Assume that the inspection is periodic, i.e. the inter-inspection time is a constant Δ.
- Suppose that the cost of repair is an increasing function $C : I \to \mathbb{R}^+$, where $C(i)$ is the repair-cost of an item at state i, and c_s is the cost of identifying a false positive item.
- Denote by K the total number of inspections, that is given by $K = \left[\dfrac{T - \tau_1}{\Delta} \right]$, where T is length of the observation period.

Costs

The aim is to derive a cost-optimal inspection strategy for such a process. The expected total cost is divided into four parts:

- $E(CR(\tau_1, \Delta))$: expected cost of repair of damaged items detected at inspection.
- $E(CFP(\tau_1, \Delta))$: expected cost associated with identifying false positives.
- $E(CI(\tau_1, \Delta))$: expected cost of inspections.
- $E(CU)$: expected cost of undetected deterioration, and can be calculated after a failure mechanism is incorporated into the model.

Expected repair cost

- In order to calculate $E(CR(\tau_1, \Delta))$, the distribution of X_{τ_j} is needed.
- The distribution can be determined using the convolution of waiting times Y_i [1] [2] .
- Assuming $\lambda_i \neq \lambda_j \ \forall i, j$, the density of the convolution is:

$$f_{Y_0+Y_1+\cdots+Y_i}(y) = \left[\prod_{k=0}^{i} \lambda_k\right] \sum_{j=0}^{i} \frac{e^{-\lambda_j y}}{\prod_{\substack{k\neq j \\ k=0}}^{i} (\lambda_k - \lambda_j)}, \quad y > 0, \quad i \geq 1.$$

- Using this density, the expected cost of repair of damaged items detected on inspection is:

$$E(CR(\tau_1, \Delta)) = \sum_{j=1}^{K} \sum_{i=0}^{4} C(i) \cdot P(X_{\tau_j} = D_i).$$

Expected total cost

- Suppose that items participate in all inspections, then the expected cost of inspection is $E(CI(\tau_1, \Delta)) = K \cdot r$, where r is the cost of a single inspection.
- The expected cost related to false positive is

$$E(CFP(\tau_1, \Delta)) = c_s \cdot \alpha \cdot \sum_{j=1}^{K} e^{-\lambda_0(\tau_j - \tau_{j-1})}.$$

- Therefore, the expected total cost to be minimized is given by:

$$E(TC(\tau_1, \Delta)) = E(CR(\tau_1, \Delta)) + E(CFP(\tau_1, \Delta)) + E(CI(\tau_1, \Delta)) + E(CU).$$

Aims

- We are currently working on implementing the model in the statistical software R.
- The expected total cost will be minimized using nonlinear minimization (*nlm* [3] [4]) in order to find optimal τ_1 and Δ.
- The next goal is to release the perfect sensitivity assumption.
- This will make the model applicable in many scenarios, e.g. disease progressions (cancer), insurance claims, ...

References

[1] Mohamed Akkouchi. On the convolution of exponential distributions. *J. Chungcheong Math. Soc*, 21(4):501–510, 2008.

[2] Markus Bibinger. Notes on the sum and maximum of independent exponentially distributed random variables with different scale parameters. *arXiv preprint arXiv:1307.3945*, 2013.

[3] John E Dennis Jr. Rb schnabel numerical methods for unconstrained optimization and nonlinear equations, 1983.

[4] Robert B Schnabel, John E Koonatz, and Barry E Weiss. A modular system of algorithms for unconstrained minimization. *ACM Transactions on Mathematical Software (TOMS)*, 11(4):419–440, 1985.

[5] Yi Yang and John Dalsgaard Sørensen. Cost-optimal maintenance planning for defects on wind turbine blades. *Energies*, 12(6):998, 2019.

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