Presentation of Individual Project

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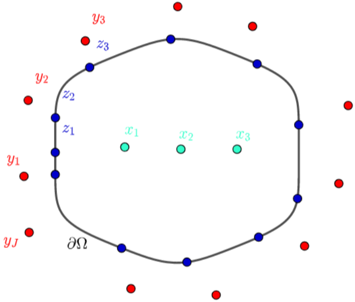
Importance of PDE's

- Describe the evolution of continuous systems
- Numerous applications in STEM, examples:
  - Transport equation: $\partial_t u + \langle v, \operatorname{grad} u \rangle = 0$, ($v \in \mathbb{R}^n$ fixed)
  - Heat (diffusion) equation: $\partial_t u - \alpha \Delta u = 0$, ($\alpha \in \mathbb{R}$ fixed)
  - Wave equation: $\frac{1}{c^2}\partial_t^2 u - \Delta u = 0$, ($c \in (0, +\infty)$ fixed)
  - Laplace and Poisson's equations: $\Delta u = f$, ($f \in L^2(\Omega)$ fixed)
  - and many more...

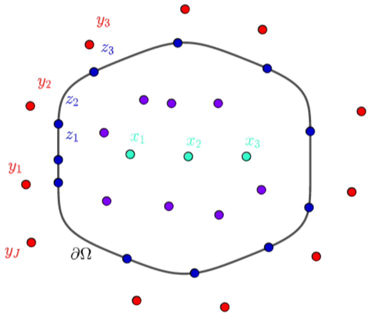
Neural Networks $\Longrightarrow$ PDE solution

[2] shows working mechanism to teach a NN to estimate the solution of Laplace's equation ($\Delta u = 0$ with a Dirichlet boundary condition) in a nontrivial $\Omega \subseteq \mathbb{R}^2$:



Now

- We try to modify the previous method to give a solution for Poisson's equation ($\Delta u = 0$ with a Dirichlet boundary condition)
- Obviously we need information from inside Ω
- Fundamental functions $\rightarrow$ Radial functions



Future

- Tuning NN
- Time dependent PDE's
- Non-linear PDE's

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